Very special relativity and Finsler geometry

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More details will be given in the paper Voigt transformations in retrospect: missed opportunities? *One more essay on the Einstein-Poincaré priority dispute* (to appear).

Einstein-Poincaré priority dispute

- Einstein doesn't cite anybody in his seminal paper
- Whittaker (1953): "Einstein published a paper which set forth the relativity theory of Poincare and Lorentz with some amplifications, and which attracted much attention"
- Modern physicists, like, for example, Logunov, find Poincaré's papers important and rise doubts about Poincaré's priority in discovery of special relativity.
- However Whittaker's conclusion, implicitly shared by many modern physicists, who had taken a trouble to indeed read Poincaré's papers, is wrong.
- It is based on the retrospective reading of Poincaré's papers. However in retrospective reading the reader can see much more than it was possible to see by contemporaries, including the author, who were bound by concrete historical context and prejudices of those days.

Einstein-Poincaré priority dispute (continued)

- If we inspect the physical literature of the founding period (1905-1918) of relativity, we will clearly see that the scientific community never hesitated in giving Einstein a due credit and there never was such a thing as Poincaré's version of relativity "accessible and perceived as such by physicists of the times"
- It is a historical fact that the relativistic revolution, as seen in the contemporary physical literature of those days, rightly or wrongly, is dominated by only one name, the Einstein's papers playing the major role, while Poincaré's ones being left virtually unnoticed
- The importance of these papers was recognized only later.
- In this respect, Poincaré-Einstein mystery "is an artefact of projecting backward a particular reading of scientific papers that does not correspond to what the actors of the time saw in them"

Citations from Y. Gingras, The collective construction of scientific memory: the Einstein-Poincaré connection and its discontents, 1905-2005, History of Science **46** (2008), 75-114.

Max Born about Einstein's paper

In 1905 Born was

"in Gottingen and well acquainted with the difficulties and puzzles encountered in the study of electromagnetic and optical phenomena in moving bodies, which we thoroughly discussed in a seminar held by Hilbert and Minkowski. We studied the recent papers by Lorentz and Poincaré, we discussed the contraction hypothesis brought forward by Lorentz and Fitzgerald, and we knew the transformations now known under Lorentz's name ...

A long time before I read Einstein's famous 1905 paper, I knew the formal mathematical side of the special theory of relativity through my teacher Hermann Minkowski. Even so, Einstein's paper was a revelation to me which had a stronger influence on my thinking than any other scientific experience... Einstein's simple consideration, by which he disclosed the epistemological root of the problem... made an enormous impression, and I think it right that the principle of relativity is connected with his name, though Lorentz and Poincaré should not be forgotten"

Poincaré about Einstein (recommendation letter, 1911)

"Monsieur Einstein is one of the most original minds I have known; in spite of his youth he already occupies a very honorable position among the leading scholars of his time. We must especially admire in him the ease with which he adapts himself to new concepts and his ability to infer all the consequences from them. He does not remain attached to the classical principles and, faced with a physics problem, promptly envisages all possibilities. This is translated immediately in his mind into an anticipation of new phenomena, susceptible some day to experimental verification. I would not say that all his expectations will resist experimental check when such checks will become possible. Since he is probing in all directions, one should anticipate, on the contrary, that most of the roads he is following will lead to dead ends; but, at the same time, one must hope that one of the directions he has indicated will be a good one; and that suffice"

Minkowski and Poincaré

- Poincaré's name was never mentioned in Minkowski's famous Cologne lecture "Raum und Zeit".
- One year before in his lecture to the Göttinger Mathematischen Gesellschaft Minkowski frequently and positively cites Poincaré.
- How can we then explain the complete disregard of Poincaré's contribution just after one year?

Max Born about Minkowski

According to Max Born's recollections, later after the Cologne lecture Minkowski told him that "it came to him as a great shock when Einstein published his paper in which the equivalence of the different local times of observers moving relative to each other was pronounced; for he had reached the same conclusions independently but did not publish them because he wished first to work out the mathematical structure in all its splendor".

- We may suppose that the discovery of the four-dimensional formalism was also a result of this process of working out the mathematical structure behind the Lorentz transformations and was made by Minkowski independently of Poincaré's 1905 papers.
- When he later realized that he had been preceded by Poincaré he needed to find reasons for downplaying Poincaré's work.
- To make the decision to exclude Poincaré's name from the Cologne lecture Minkowski needed some serious reason to psychologically justify such an unfair omission.

Another oddity of the Cologne lecture

- In the Cologne lecture Minkowski never mentions Lorentz transformations (so named by Poincaré), instead he refers to transformations of the group G_c.
- On December 21, 1907, Minkowski talked to the Göttingen scientific society the text of which, with all results of the future Cologne lecture presented with great details, was published in April 1908— a beginning of his downplaying of the Poincaré's contribution.
- Overall impression that the reader could infer from this historically important publication might be that Minkowski is "suggesting that the main (if not the only) contribution by Poincaré is to have given the Lorentz transformations . . . their name" (M. M. Capria, arXiv:1111.7126).

Voigt a missing link?

The reason for suppression of the name "Lorentz transformations" in the Cologne lecture is unknown, but very probably it was linked to Minkowski's discovery that essential application of the central role the Lorentz symmetry plays in optics goes back to Woldemar Voigt's 1887 paper.

- Although Voigt and Lorentz were in correspondence since 1883, Lorentz was unaware of Voigt's Doppler principle paper and it was not until 1908 that Voigt sent him a reprint of this paper.
- Voigt and Minkowski were friends at Göttingen. So it is possible that Voigt informed Minkowski about his correspondence with Lorentz in around July, 1908 and probably that's how Minkowski became aware of the Voigt's 1887 Doppler principle paper.
- Minkowski cites (incorrectly) Voigt in the Cologne lecture as the discoverer of the Lorentz symmetry.
- "Historically, I want to add that the transformation, which play the main role in the relativity principle, were first mathematically discussed by Voigt in the year 1887."

Voigt a missing link? (continued)

- Convincing himself that it was Voigt who should be credited for apprehension of the central role of the Lorentz symmetry, perhaps, it was psychologically more easy for Minkowski to decide to omit Poincaré's name from the Cologne lecture.
- Very likely, this pernicious decision was further eased by Poincaré's style of writing who "habitually wrote in a self-effacing manner. He named many of his discoveries after other people, and expounded many important and original ideas in writings that were ostensibly just reviewing the works of others, with 'minor amplifications and corrections'. Poincaré's style of writing, especially on topics in physics, always gave the impression that he was just reviewing someone else's work"

L. Pyenson, Physics in the shadow of mathematics: The Göttingen electron-theory seminar of 1905, Arch. Hist. Ex. Sci. **21** (1979), 55-89.

$$\square\,\Phi(x,y,z,t)=\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\Delta\right)\Phi(x,y,z,t)=0,$$

$$\begin{aligned}
\xi &= m_1(V)\,x+n_1(V)\,y+p_1(V)\,z-\alpha_0(V)\,t,\\
\eta &= m_2(V)\,x+n_2(V)\,y+p_2(V)\,z-\beta_0(V)\,t,\\
\zeta &= m_3(V)\,x+n_3(V)\,y+p_3(V)\,z-\gamma_0(V)\,t,\\
\tau &= t-\left[a_0(V)\,x+b_0(V)\,y+c_0(V)\,z\right],
\end{aligned}$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial \tau^2}-\frac{\partial^2}{\partial \xi^2}-\frac{\partial^2}{\partial \eta^2}-\frac{\partial^2}{\partial \zeta^2}\right)\Phi(\xi,\eta,\zeta,\tau)=0,$$

"as it must be"

$$\Box' = \frac{1}{c^2}\frac{\partial^2}{\partial\tau^2} - \frac{\partial^2}{\partial\xi^2} - \frac{\partial^2}{\partial\eta^2} - \frac{\partial^2}{\partial\zeta^2} = \gamma^2(V)\,\Box = \gamma^2(V)\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}\right),$$

$$\gamma^2\left[1-(a_0^2+b_0^2+c_0^2)c^2\right]=1, \quad \gamma^2\left[(m_1^2+n_1^2+p_1^2)-\frac{V^2}{c^2}\right]=1,$$
$$\gamma^2\left[m_2^2+n_2^2+p_2^2\right]=1, \quad \gamma^2\left[m_3^2+n_3^2+p_3^2\right]=1.$$

$$m_1\,a_0+n_1\,b_0+p_1\,c_0-\frac{V}{c^2}=0, \quad m_2\,a_0+n_2\,b_0+p_2\,c_0=0, \quad m_3\,a_0+n_3\,b_0+p_3\,c_0=0.$$

$$m_1\,m_2+n_1\,n_2+p_1\,p_2=0, \quad m_1\,m_3+n_1\,n_3+p_1\,p_3=0, \quad m_2\,m_3+n_2\,n_3+p_2\,p_3=0.$$

Voigt transformations

We have thirteen unknowns (including γ^2) and ten equations. So, concludes Voigt, three of them can be chosen arbitrarily. The natural choice is $m_1 = 1$, $n_1 = 0$, $p_1 = 0$, which makes the transformation law for ξ identical to the Galilean transformation.

$$\begin{aligned}
\xi &= x - V\,t, \\
\eta &= \gamma^{-1}\,y, \\
\zeta &= \gamma^{-1}\,z, \\
\tau &= t - \tfrac{V}{c^2}\,x.
\end{aligned}$$

The inverse transformations have the form

$$\begin{aligned}
x &= \gamma^2\,(\xi + V\,\tau)\,, \\
y &= \gamma\,\eta, \\
z &= \gamma\,\zeta, \\
t &= \gamma^2\left(\tau + \tfrac{V}{c^2}\,\xi\right).
\end{aligned}$$

Are Voigt and Lorentz transformations equivalent?

The answer depends on the reading of Voigt transformations: modern reader can see quite different contexts in them compared to Voigt's contemporaries.

- The relativistic reading of Voigt transformations is certainly possible.
- The observer in the moving frame uses a standard atomic clock of the æther frame to define a time unit.
- The resulting theory will be completely equivalent (although perhaps less convenient) to special relativity.

$$x' = x - V\,t,\;\; y' = y,\;\; z' = z,\;\; t' = t.$$

This form of Galilean transformations assumes Newtonian absolute time. However, if the velocity V of S' with respect to the æther frame S is less than light velocity c, it is possible to perform Poincaré-Einstein synchronization of clocks in S' and as a result we get another parametrization of the Galilean space-time in S'.

$$x' = x - V\,t,\;\; y' = y,\;\; z' = z,\;\; t' = \gamma^2\left(t - \frac{V}{c^2}x\right).$$

Zahar transformations. E. Zahar, Mach, Einstein, and the Rise of Modern Science, Brit. J. Phil. Sci. **28** (1977), 195-213.

Non-relativistic limit of Lorentz transformations

- Contrary to a prevailing belief, the non-relativistic limit of Lorentz transformations, when $\beta = V/c \ll 1$, is not the Galilean transformations but the Zahar transformations in which β^2 terms are neglected.
- The Galilean limit additionally requires $\beta \ll \frac{ct}{x}, \quad \frac{x}{ct} \sim \beta$ which is not necessarily true if the spatial separation x between two events is comparable or larger than temporal separation ct.
- The reason for this mismatch of non relativistic limits is the use of different synchronization conventions in Lorentz transformations and Galilean transformations: for sufficiently distant events in Minkowski world one cannot ignore not absolute nature of distant simultaneity.

Tangherlini transformations

- Absolute simultaneity doesn't contradict special relativity — Tangherlini's 1958 PhD dissertation supervised by Sidney Drell and Donald Yennie (at the initial stage of the work).
- External synchronization.
- Tangherlini transformations:

$$x' = \gamma(x - Vt), \quad y' = y, \quad z' = z, \quad t' = \gamma^{-1} t.$$

- Non-relativistic limit of Tangherlini transformations are Galilean transformations.
- As was shown by Kretschmann already in 1917, any space-time theory can be expressed in general covariant manner, not only general relativity.

F. R. Tangherlini, The Velocity of Light in Uniformly Moving Frame, Abraham Zelmanov J. **2** (2009), 44-110.

- Suppose two distant clocks clocks A and B are motionless in a common inertial frame S. If a light signal is sent from A at time t_A, which is instantaneously reflected by B at time t_B, and arrives back at A at time t'_A, then A and B are Poincaré-Einstein synchronized if $t_B - t_A = t'_A - t_B$, or

$$t_B = \frac{1}{2}(t_A + t'_A) = t_A + \frac{1}{2}(t'_A - t_A).$$

- Reichenbach modifies this definition of synchronization as follows

$$t_B = t_A + \epsilon(t'_A - t_A).$$

- where now ϵ is some real parameter and $0 \leq \epsilon \leq 1$ (because for causality reasons we need $t_B \geq t_A$ and $t_B \leq t'_A$, the equality corresponding to the infinite one-way velocity of the light signal. In fact these limiting cases were excluded by Reichenbach).

If ϵ-synchronization is adopted in the reference frame S, while ϵ'-synchronization in the reference frame S', Winnie transformations have the form

$$\begin{aligned} x' &= \frac{x-Vt}{\alpha}, \\ y' &= y, \\ z' &= z, \\ t' &= \frac{1}{\alpha}\left\{[1+2\beta(1-\epsilon-\epsilon')]\,t-[2(\epsilon-\epsilon')+4\beta\epsilon(1-\epsilon)]\,\frac{x}{c}\right\}, \end{aligned}$$

where

$$\alpha=\sqrt{[1-(2\epsilon-1)\beta]^2-\beta^2}.$$

J. A. Winnie, Special Relativity without One-Way Velocity Assumptions: Part II, Phil. Sci. **37**, (1970), 223-238.

Special cases of Winnie transformations

- If $\epsilon = \epsilon' = 1/2$, we recover the ordinary Lorentz transformations.
- when $\epsilon = 1/2$ and $\epsilon' = (1+\beta)/2$, we get Tangherlini transformations.
- $\epsilon = \epsilon' = 0$ — another choice of the coordinate chart in the Minkowski space-time that corresponds to absolute simultaneity ('everyday' clock synchronization).

C. Leubner, K. Aufinger and P. Krumm, Elementary relativity with 'everyday' clock synchronization, Eur. J. Phys. **13** (1992), 170-177.

Let's go back to the beginnings

"Everything has been said before, but since nobody listens we have to keep going back and beginning all over again."

André Gide

Derivation of Lorentz transformations

Under the assumption that measuring rods do not change their lengths when gently set into a state of uniform motion, the Galilean transformations $x' + Vt = x$ are simply a statement that the length of a finite interval is an additive quantity: the length of an union of two intervals equals to the sum of their lengths.

- in 1988 Oliver Heaviside showed that the electric field of a charge in motion relative to the æther is no longer spherically symmetric and becomes distorted in the the longitudinal direction.
- FitzGerald contraction hypothesis, 1889.

$$x = k_1(V)x' + Vt, \quad x' = k_2(-V)x - Vt'$$

- Relativity principle $\rightarrow k_1(V) = k_2(V)$.
- Spatial isotropy $\rightarrow k_1(-V) = k_1(V)$.

$$x' = \frac{1}{k_1(V)}(x - Vt), \quad x = \frac{1}{k_2(-V)}(x' + Vt').$$

$$t' = \frac{1}{k_1(V)}\left[t - \frac{1 - K(V)}{V}x\right], \quad t = \frac{1}{k_2(-V)}\left[t' + \frac{1 - K(V)}{V}x'\right],$$

where $K(V) = k_1(V)k_2(-V)$.

Derivation of Lorentz transformations (continued)

Therefore we get the transformation

$$
\begin{aligned}
x' &= \frac{1}{k_1(V)}\,(x - V\,t),\\
y' &= \lambda(V)y,\\
z' &= \lambda(V)z,\\
t' &= \frac{1}{k_1(V)}\,\left[t - \frac{1-K(V)}{V}\,x\right],
\end{aligned}
$$

and its inverse

$$
\begin{aligned}
x &= \frac{1}{k_2(-V)}\,(x' + V\,t'),\\
y &= \lambda^{-1}(V)y',\\
z &= \lambda^{-1}(V)z',\\
t &= \frac{1}{k_2(-V)}\,\left[t' + \frac{1-K(V)}{V}\,x'\right].
\end{aligned}
$$

Velocity addition rule:

$$v_x = \frac{v'_x + V}{1 + \frac{1-K(V)}{V}\, v'_x} = F(v'_x, V),$$

$$v_y = \frac{k_2(-V)\, v'_y}{\lambda(V)\left[1 + \frac{1-K(V)}{V}\, v'_x\right]},$$

$$v_z = \frac{k_2(-V)\, v'_z}{\lambda(V)\left[1 + \frac{1-K(V)}{V}\, v'_x\right]}.$$

Universality of the light velocity will demand

$$K(V)=1-\frac{V^2}{c^2}=\frac{1}{\gamma^2},$$

and

$$\frac{k_2(-V)}{\lambda(V)}=\frac{1}{\gamma}.$$

Therefore

$$\lambda(V)=\frac{k_2(-V)}{\sqrt{K(V)}}=\sqrt{\frac{k_2(-V)}{k_1(V)}},$$

$$\frac{1}{k_1(V)}=\sqrt{\frac{k_2(-V)}{k_1(V)}}\,\frac{1}{\sqrt{k_1(V)k_2(-V)}}=\frac{\lambda(V)}{\sqrt{K(V)}}=\lambda(V)\gamma.$$

λ-Lorentz transformations

$$
\begin{aligned}
x' &= \lambda(V)\gamma\,(x - V\,t)\,, \\
y' &= \lambda(V)y, \\
z' &= \lambda(V)z, \\
t' &= \lambda(V)\gamma\,\left(t - \frac{V}{c^2}\,x\right).
\end{aligned}
$$

Both Einstein and Poincarë got these λ-Lorentz transformations and then both argued that $\lambda(V) = 1$.

- Group property $\rightarrow \lambda(V_1 \oplus V_2) = \lambda(V_1)\lambda(V_2)$
- In particular $\lambda(V)\lambda(-V) = \lambda(0) = 1$
- Spatial isotropy $\rightarrow \lambda(-V) = \lambda(V)$
- In combination with the positivity of $\lambda \rightarrow \lambda(V) = 1$.

Cauchy exponential functional equation

$$\tanh\psi = \beta = \frac{V}{c}.$$

$$\lambda(\psi_1+\psi_2) = \lambda(\psi_1)\,\lambda(\psi_2).$$

Although there exist infinitely many wildly discontinuous solutions, the continuous solutions, which are the only ones acceptable in the context of λ-Lorentz transformations, all have the form

$$\lambda(\psi) = e^{-b\psi} = \left(\frac{1-\beta}{1+\beta}\right)^{b/2},$$

$$
\begin{aligned}
x' &= \left(\frac{1-\beta}{1+\beta}\right)^{b/2} \gamma\,(x - V\,t)\,,\\
y' &= \left(\frac{1-\beta}{1+\beta}\right)^{b/2} y,\\
z' &= \left(\frac{1-\beta}{1+\beta}\right)^{b/2} z,\\
t' &= \left(\frac{1-\beta}{1+\beta}\right)^{b/2} \gamma\,\left(t - \frac{V}{c^2}\,x\right).
\end{aligned}
$$

V. Lalan, Sur les postulats qui sont á la base des cinématiques, Bull. Soc. Math. Fr. **65** (1937), 83-99.

Light-cone coordinates $u = ct + x, \quad v = ct - x.$

$$u' = e^{-(1+b)\psi}\, u, \quad v' = e^{(1-b)\psi}\, v, \quad y' = e^{-b\psi}\, y, \quad z' = e^{-b\psi}\, z.$$

Invariant quantities

$$\left(\frac{v'}{u'}\right)^{b} u'v' = \left(\frac{v}{u}\right)^{b} uv, \quad \left(\frac{v'}{u'}\right)^{b} y'^{\,2} = \left(\frac{v}{u}\right)^{b} y^{2}, \quad \left(\frac{v'}{u'}\right)^{b} z'^{\,2} = \left(\frac{v}{u}\right)^{b} z^{2}, \quad \frac{u'v'}{y'^{\,2}} = \frac{uv}{y^{2}}, \quad \frac{u'v'}{z'^{\,2}} = \frac{uv}{z^{2}}.$$

a generalization of the relativistic interval:

$$s^2 = \left(\frac{v}{u}\right)^{b} \left(uv - y^2 - z^2\right) \left(\frac{uv}{uv - y^2 - z^2}\right)^{b} = v^{2b}(uv - y^2 - z^2)^{1-b} = (ct - x)^{2b} \left(c^2t^2 - x^2 - y^2 - z^2\right)^{1-b}.$$

$$ds^2 = (c\,dt - dx)^{2b} \left(c^2\,dt^2 - dx^2 - dy^2 - dz^2\right)^{1-b} = (n_\nu\, dx^\nu)^{2b} \, (dx_\mu\, dx^\mu)^{1-b},$$

where $n^\mu = (1,1,0,0) = (1,\vec{n})$, $\vec{n}^{\,2} = 1$, is the fixed null-vector defining a preferred null-direction in the space-time.

Lalan-Alway-Bogoslovsky transformations

$$\left(n_\nu dx'^\nu\right)^{2b}\left(dx'_\mu dx'^\mu\right)^{1-b} = \left(n_\nu dx^\nu\right)^{2b}\left(dx_\mu dx^\mu\right)^{1-b}.$$

$$x'^\mu = D(\lambda) R^\mu_\nu(\vec{m};\alpha)\, L^\nu_\sigma(\vec{V}) x^\sigma .$$

$$D(\lambda) = \left[\gamma(1-\vec{\beta}\cdot\vec{n})\right]^b$$

$$\vec{m} = \frac{\vec{n}\times\vec{\beta}}{|\vec{n}\times\vec{\beta}|}$$

$$\cos\alpha = 1 - \frac{\gamma-1}{\gamma\beta^2}\,\frac{[\vec{n}\times\vec{\beta}]^2}{1-\vec{n}\cdot\vec{\beta}} = 1 - \frac{\gamma}{\gamma+1}\,\frac{[\vec{n}\times\vec{\beta}]^2}{1-\vec{n}\cdot\vec{\beta}}.$$

$$x_0' = \left[\gamma(1-\vec{\beta}\cdot\vec{n})\right]^{b}\gamma\,(x_0-\vec{\beta}\cdot\vec{r}),$$
$$\vec{r}\,' = \left[\gamma(1-\vec{\beta}\cdot\vec{n})\right]^{b}\left\{\vec{r}-\frac{\vec{\beta}\,(x_0-\vec{n}\cdot\vec{r})}{1-\vec{\beta}\cdot\vec{n}}-\vec{n}\left[\gamma\,\vec{\beta}\cdot\vec{r}+\frac{\gamma-1}{\gamma}\,\frac{\vec{n}\cdot\vec{r}}{1-\vec{\beta}\cdot\vec{n}}+\frac{(\gamma-1)\vec{\beta}\cdot\vec{n}-\gamma\beta^2}{1-\vec{\beta}\cdot\vec{n}}\,x_0\right]\right\}.$$

If $\vec{\beta}\perp\vec{n}$, $\quad x\parallel\vec{\beta}$, $\quad y\parallel\vec{n}$:

$$\begin{aligned}
x' &= \gamma^{b}\left[x-Vt+\beta\,y\right],\\
y' &= \gamma^{1+b}\left[(1-\beta^2)\,y-\beta\,(x-Vt)\right],\\
z' &= \gamma^{b}\,z,\\
t' &= \gamma^{1+b}\left[t-\frac{V}{c^2}\,x\right].
\end{aligned}$$

Lalan-Alway-Bogoslovsky transformations (continued)

G. Alway, Generalization of the Lorentz Transformation, Nature **224** (1969), 155-156.

J. Strnad, Generalization of the Lorentz Transformation, Nature **226** (1970), 137-138.

G. Yu. Bogoslovsky, On a special relativistic theory of anisotropic space-time, Dokl. Akad. Nauk SSSR Ser. Fiz. **213** (1973), 1055-1058 (in Russian).

G. Yu. Bogoslovsky, A special-relativistic theory of the locally anisotropic space-time. I: The metric and group of motions of the anisotropic space of events, Nuovo Cim. B **40** (1977), 99-115.

Very special relativity

- Lorentz group has no 5-parameter subgroup and only one, up to isomorphism, 4-parameter subgroup, namely SIM(2).
- Very special relativity: Lorentz group $\rightarrow$ SIM(2), Poincaré group $\rightarrow$ ISIM(2).
- Very special relativity breaks Lorentz symmetry in a very mild and minimal way.
- Either P, T or CP discrete symmetries enlarges SIM(2) subgroup to the full Lorentz group.
- The existence of the preferred light-like direction n^μ can be interpreted as the existence of light-like æther. Difficult to detect: it doesn't single out any preferred inertial reference frame.
- Since CP violating effects are small, Lorentz-violating effects in very special relativity are expected to be also small.

A. G. Cohen and S. L. Glashow, Very special relativity, Phys. Rev. Lett. **97** (2006), 021601.

If $b = 0$, VSR is unnatural

VSR is equivalent to $b = 0$ and preferred light-like direction n^μ

- When $b = 0$, that is when the space is isotropic, we have no reason to introduce the preferred light-like direction n^μ.
- Of course we can do this artificially and consequently arrive at the generalized Lorentz transformations instead of usual Lorentz transformations.
- However in this case $\vec{n}$ has no physical meaning and just serves to calibrate the orientations of space axes of the inertial frames of reference in such a way that if in one such frame of reference the ray of light has the direction $\vec{n}$, it will have the same direction in all inertial reference frames.
- The resulting theory will be equivalent to special relativity if we can choose n^μ arbitrarily and all such choices are equivalent.
- In fact it suffices to require that the choices $\vec{n}$ and $-\vec{n}$ are equivalent.

A symmetry group G can be contracted towards its continuous subgroup S, which remain intact under the contraction process.

- Lie algebra of G

$$[J_i, J_j] = f^{(1)}_{ijk} J_k, \quad [I_i, J_j] = f^{(2)}_{ijk} J_k + g^{(2)}_{ijk} I_k, \quad [I_i, I_j] = f^{(3)}_{ijk} J_k + g^{(3)}_{ijk} I_k.$$

- J_i are generators of S, I_i are remaining generators. Change of basis

$$J'_i = J_i, \quad I'_i = \epsilon\, I_i,$$

- produces a new commutation relations

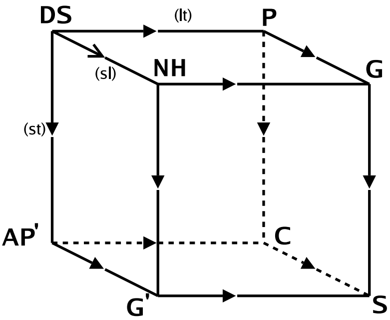
$$[J'_i, J'_j] = f^{(1)}_{ijk} J'_k, \quad [I'_i, J'_j] = \epsilon f^{(2)}_{ijk} J'_k + g^{(2)}_{ijk} I'_k, \quad [I'_i, I'_j] = \epsilon^2 f^{(3)}_{ijk} J'_k + \epsilon g^{(3)}_{ijk} I'_k.$$

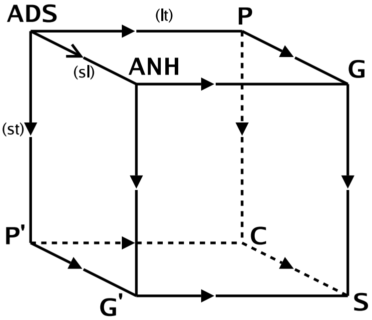
- When $\epsilon \to 0$, the base change becomes singular but the commutation relations still have a well-defined limit

$$[J'_i, J'_j] = f^{(1)}_{ijk} J'_k, \quad [I'_i, J'_j] = g^{(2)}_{ijk} I'_k, \quad [I'_i, I'_j] = 0.$$

  and define in general another symmetry group G'.

E. Inönü and E. P. Wigner, On the Contraction of groups and their represenations, Proc. Nat. Acad. Sci. **39** (1953), 510-524.





H. Bacry and J. Lévy-Leblond, Possible kinematics, J. Math. Phys. **9** (1968), 1605-1614.
Z. K. Silagadze, Relativity without tears, Acta Phys. Polon. B **39** (2008), 811-885.

- true physical theory should be stable against small deformations of its underlying algebraic (group) structure.
- Lie algebra of inhomogeneous Galilei group is not stable and its deformation leads to Lie algebra of the Poincaré group. Consequently the relativity theory based on the Poincaré group has a greater range of validity than Galilean relativity.
- However the Poincaré Lie algebra is by itself unstable and its deformation leads to either de Sitter or anti-de Sitter Lie algebras. Therefore it is not surprising that the cosmological constant turned out to be not zero and correspondingly the asymptotic vacuum space-time is not Minkowski but de Sitter space-time.
- What is really surprising is why the cosmological constant is so small that makes special relativity valid for all practical purposes.

I. E. Segal, A class of operator algebras which are determined by groups, Duke Math. J. **18** (1951), 221-265.

Lie algebra of the very special relativity symmetry group ISIM(2) is not stable against small deformations of its structure and a physically relevant deformation, $\mathrm{DISIM}_b(2)$, of it does exist. Therefore, in light of Segal's principle, we expect that the very special relativity cannot be a true symmetry of nature and should be replaced by $\mathrm{DISIM}_b(2)$ and the corresponding Finslerian space-time. Drawing an analogy with the cosmological constant, it can be argued that b is really not zero, but very small.

G. W. Gibbons, J. Gomis and C. N. Pope, General very special relativity is Finsler geometry, Phys. Rev. D **76** (2007), 081701.

- Finsler metric of the Lalan-Alway-Bogoslovsky type is a very natural generalization of special relativity, most likely indeed realized in nature.
- The anisotropy parameter b is expected to be very small but nonzero.
- In this case, to detect the effects of Finslerian nature of space-time in laboratory experiments will be almost impossible.
- Nevertheless, the question of the true value of the parameter b has the same fundamental significance as the question why the cosmological constant is so small.
- Perhaps both questions are just different parts of the same mystery.