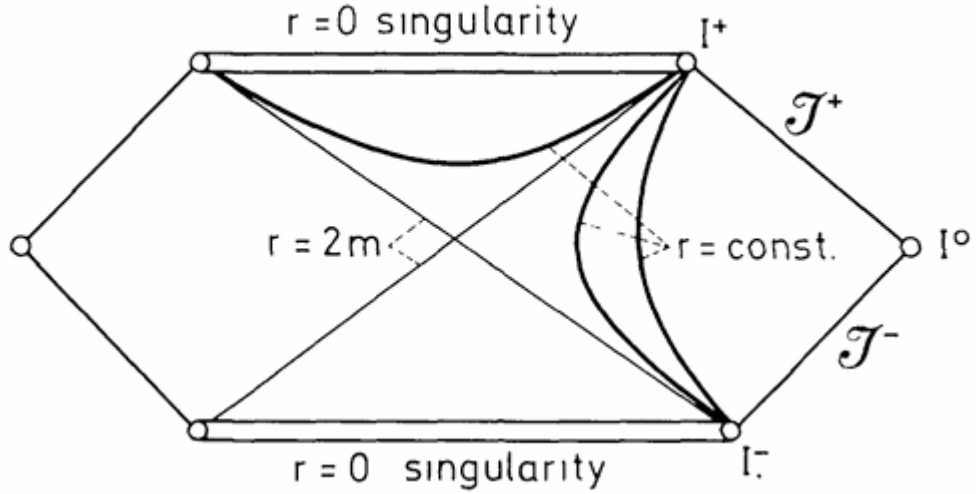
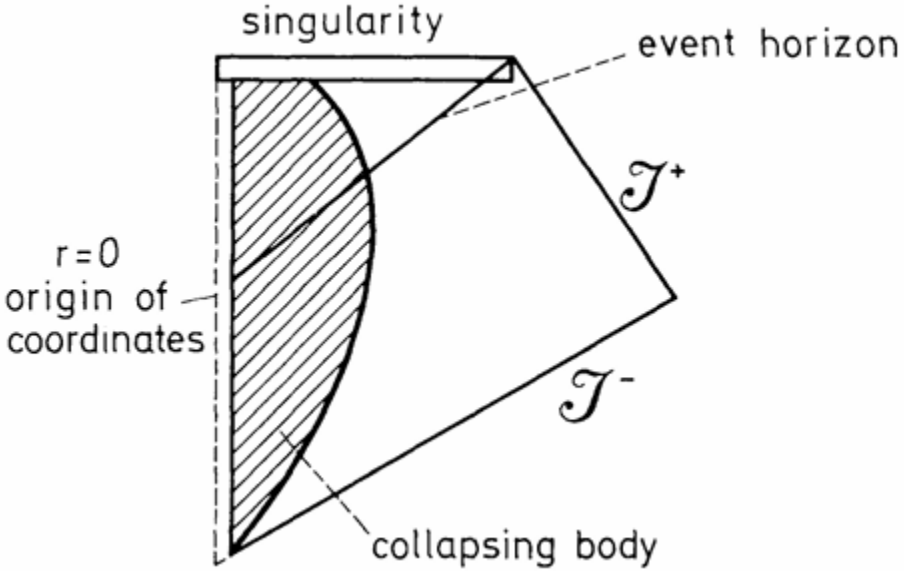
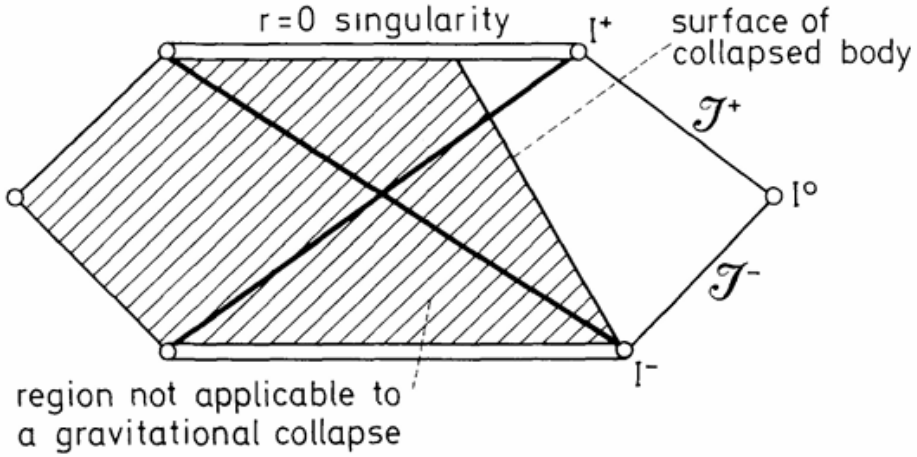
ჰოკინგის რადიაცია

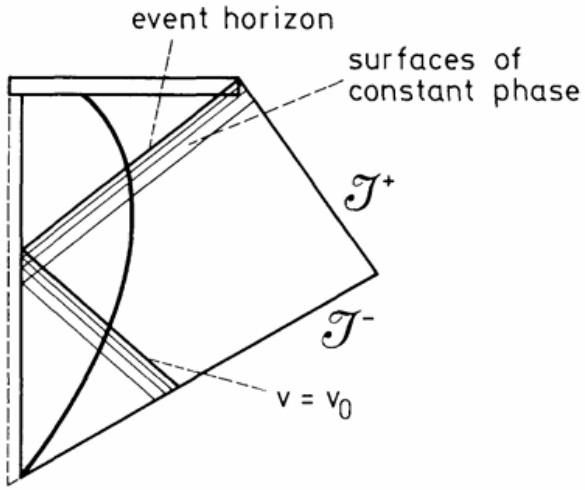
ლევან ლოლაძე

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \left(1-\frac{2M}{r}\right)^{-1}dr^2 + r^2(d\theta^2+\sin^2\theta d\phi^2)\,.$$









$$\phi_{;ab}g^{ab}=0\,.$$

$$\phi=\sum_i\{f_i a_i+\bar{f}_i a_i^\dagger\}\,.$$

$$\tfrac{1}{2}i\int_S(f_i\bar{f}_{j;a}-\bar{f}_j f_{i;a})d\Sigma^a=\delta_{ij}$$

$$\phi=\sum_i\{p_i b_i+\bar{p}_i b_i^\dagger+q_i c_i+\bar{q}_i c_i^\dagger\}\,.$$

$$p_i=\sum_j(\alpha_{ij}f_j+\beta_{ij}\bar{f}_j)\,,$$

$$q_i=\sum_j(\gamma_{ij}f_j+\eta_{ij}\bar{f}_j)\,.$$

$$b_i=\sum_j(\bar{\alpha}_{ij}a_j-\bar{\beta}_{ij}a_j^\dagger)\,,$$

$$c_i=\sum_j(\bar{\gamma}_{ij}a_j-\bar{\eta}_{ij}a_j^\dagger)\,.$$

$$a_i|0\rangle=0 \quad \text{for all } i\,.$$

$$\langle 0_-|b_i^\dagger b_i|0_-\rangle=\sum_j|\beta_{ij}|^2\,.$$

$$f_{\omega' lm}=(2\pi)^{-\frac{1}{2}}r^{-1}(\omega')^{-\frac{1}{2}}F_{\omega'}(r)e^{i\omega' v}Y_{lm}(\theta,\phi)\,,$$

$$p_{\omega lm}=(2\pi)^{-\frac{1}{2}}r^{-1}\omega^{-\frac{1}{2}}P_\omega(r)e^{i\omega u}Y_{lm}(\theta,\phi)\,,$$

$$v=t+r+2M\log\left|\frac{r}{2M}-1\right|,$$

$$u=t-r-2M\log\left|\frac{r}{2M}-1\right|.$$

$$p_\omega=\int_0^\infty(\alpha_{\omega\omega'}f_{\omega'}+\beta_{\omega\omega'}\bar{f}_{\omega'})d\omega'\,.$$

$$\lambda = -Ce^{-\kappa u}$$

$$n^a = DK^a .$$

$$-\frac{\omega}{\kappa}(\log(v_0 - v) - \log D - \log C) .$$

$$p_\omega^{(2)} \sim (2\pi)^{-\frac{1}{2}} \omega^{-\frac{1}{2}} r^{-1} P_\omega^- \exp\left(-i\frac{\omega}{\kappa}\left(\log\left(\frac{v_0 - v}{CD}\right)\right)\right)$$

$$\alpha^{(2)}_{\omega\omega'} \approx (2\pi)^{-1} P_\omega^- (CD)^{\frac{i\omega}{\kappa}} \exp(i(\omega - \omega')v_0)\left(\frac{\omega'}{\omega}\right)^{\frac{1}{2}} \Gamma\left(1 - \frac{i\omega}{\kappa}\right)(-i\omega')^{-1+\frac{i\omega}{\kappa}},$$

$$\beta^{(2)}_{\omega\omega'} \approx -i\alpha^{(2)}_{\omega(-\omega')} .$$

$$|\alpha^{(2)}_{\omega\omega'}| = \exp\left(\frac{\pi\omega}{\kappa}\right)|\beta^{(2)}_{\omega\omega'}| .$$

$$|\alpha^{(2)}_{jn\omega'}| = \exp(\pi\omega\kappa^{-1})|\beta^{(2)}_{jn\omega'}| .$$

$$|\alpha_{jn\omega'}| = \left|(2\pi)^{-1} P_\omega^- \omega^{-\frac{1}{2}} \Gamma\left(1 - \frac{i\omega}{\kappa}\right)\varepsilon^{-\frac{1}{2}}(\omega')^{-\frac{1}{2}} \cdot \int_{j\varepsilon}^{(j+1)\varepsilon} \exp i\omega''(-2\pi n\varepsilon^{-1} + \kappa^{-1}\log\omega')d\omega''\right|$$

$$= \left|\pi^{-1} P_\omega^- \omega^{-\frac{1}{2}} \Gamma\left(1 - \frac{i\omega}{\kappa}\right)\varepsilon^{-\frac{1}{2}}(\omega')^{-\frac{1}{2}} z^{-1} \sin\tfrac{1}{2}\varepsilon z\right|$$

$$\int_0^\infty |\beta_{jn\omega'}|^2 d\omega' .$$

$$\Gamma_{jn} = \int_0^\infty (|\alpha^{(2)}_{jn\omega'}|^2 - |\beta^{(2)}_{jn\omega'}|^2)d\omega'$$

$$p_{jn} = \varepsilon^{-\frac{1}{2}} \int_{j\varepsilon}^{(j+1)\varepsilon} e^{-2\pi i n\varepsilon^{-1}\omega} p_\omega d\omega$$

$$p_{jn} = \int_0^\infty (\alpha_{jn\omega'} f_{\omega'} + \beta_{jn\omega'} \bar{f}_{\omega'})d\omega'$$

$$\Gamma_{jn}(\exp(2\pi\omega\kappa^{-1}) - 1)^{-1} .$$

შავი ხვრელის სიცოცხლის ხანგრძლივობა

$$T_{BH} = \frac{1}{8\pi G M_{BH}} = \frac{1}{4\pi r_s} ,$$

$$S_{BH} = 2\pi M_{BH} r_s ,$$

$$A = 27\pi r_s^2 .$$

$$\frac{d\dot{N}_i}{dQ} = \frac{\sigma_s}{8\pi^2} \frac{Q^2}{e^{Q/T_{BH}} - (-1)^{2s}}$$

$$\dot{N}_i = \frac{27\Gamma_s T_{BH}}{128\pi^3} \int \frac{u^2}{e^u - (-1)^{2s}} du .$$

$$\int_0^\infty \frac{z^{n-1}}{e^z - 1} dz = \Gamma(n)\zeta(n), \quad \int_0^\infty \frac{z^{n-1}}{e^z + 1} dz = \frac{1}{2^n}(2^n - 2)\Gamma(n)\zeta(n),$$

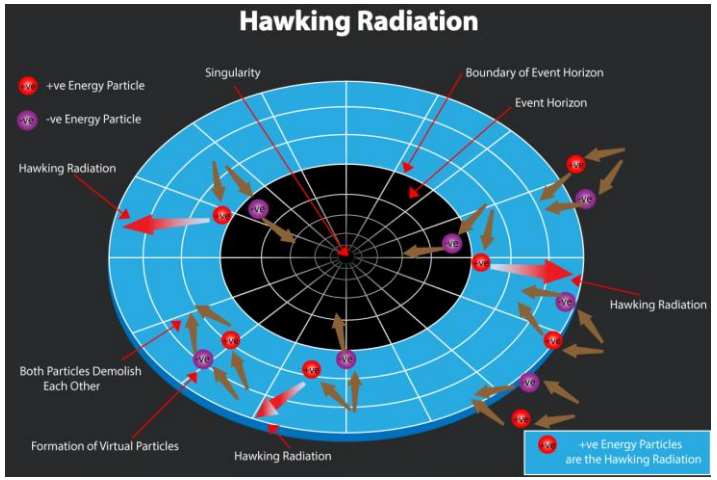
$$\dot{N}_i = f \frac{27\Gamma_s}{128\pi^3} \Gamma(3)\zeta(3) T_{\rm BH} ,$$

$$\dot{N}_i \approx B \times 10^{20} \left(\frac{T_{BH}}{GeV} \right) s^{-1} ,$$

$$\frac{dM_{BH}}{dt} = 8.3 \times 10^{73} \frac{GeV^4}{M_{BH}^2} .$$

$$\dot{M}_{BH} = -\sum_i c_i \tilde{f} \frac{27\Gamma_s}{128\pi^3} \Gamma(4)\zeta(4) T_{BH}^2 ;$$

$$\frac{\dot{M}_{BH}}{dQ} = -\sum_i c_i \frac{\sigma_s}{8\pi^2} \frac{Q^3}{e^{Q/T_{BH}} - (-1)^{2s}} ,$$



$$\tau_{BH} = 1.2 \times 10^{-74} \frac{1}{GeV^4} \int M_{BH}^2 dM_{BH} \simeq 1.6 \times 10^{-26} \frac{M_{BH}^3 yr}{kg^3} .$$

გამოყენებული ლიტერატურა

- Notes on the Hawking Effect Jeremy Butterfield and Bryan W Roberts- https://personal.lse.ac.uk/robert49/teaching/partiii/pdf/NoteOnHawking.pdf
- Particle Creation by Black Holes S. W. Hawking

გმადლობთ ყურადღებისთვის