Sterile neutrino
and
Seesaw Mechanism

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Dirac and Majorana mass

Two distinct classes of neutrino mass terms are allowed in the Lagrangian of electroweak interactions. These are called **Dirac** and **Majorana** mass terms.

The Dirac mass terms, have form :

$$L_D = -m_D(\nu_R \nu_L + \nu_L \nu_R) \tag{1}$$

The Majorana mass terms, have form :

$$L_M = -\frac{1}{2} m_M^L \left(\overline{\nu_L} \nu_L^c + \overline{\nu_L^c} \nu_L\right) - \frac{1}{2} m_M^R \left(\overline{\nu_R} \nu_R^c + \overline{\nu_R^c} \nu_R\right) \tag{2}$$

where sub/superscripts L and R designate left or right hand chirality, and the superscript c represents charge conjugation

Mass Matrix M

The most general renormalizable Lagrangian for the neutrino masses includes both the Dirac and Majorana mass terms

We can express (1) and (2) in terms of a mass matrix M as:

$$\mathcal{L}_{\text{mass terms}} = -\frac{1}{2}\begin{pmatrix}\overline{\nu_L} & \overline{\nu_R^c}\end{pmatrix}\mathcal{M}\begin{pmatrix}\nu_L^c \\ \nu_R\end{pmatrix} + \text{h.c.} \tag{3}$$

Where:

$$\mathcal{M} = \begin{pmatrix} m_M^L & m_D \\ m_D & m_M^R \end{pmatrix} \tag{4}$$

Including h.c. term :

$$\mathcal{L}_{\text{mass terms}} = -\frac{1}{2}\begin{pmatrix}\overline{\nu_L} & \overline{\nu_R^c}\end{pmatrix}\mathcal{M}\begin{pmatrix}\nu_L^c \\ \nu_R\end{pmatrix} - \frac{1}{2}\begin{pmatrix}\overline{\nu_L^c} & \overline{\nu_R}\end{pmatrix}\mathcal{M}\begin{pmatrix}\nu_L \\ \nu_R^c\end{pmatrix},$$

Eigenvalues Of Mass Matrix M

With the Lagrangian including Dirac and Majorana mass terms, the masses of the physical neutrino states are the **eigenvalues of mass matrix M**

These can be found from the characteristic equation det $(M - \lambda I) = 0$

$$(m_M^L - \lambda)(m_M^R - \lambda) - (m_D)^2 = 0 \tag{5}$$

If we solve (5) we get:

$$m_{1,2} = \lambda_{1,2} = \frac{(m_M^R + m_M^L) \pm \sqrt{(m_M^R + m_M^L)^2 - 4(m_M^R m_M^L - m_D^2)}}{2} \tag{6}$$

- For $\lambda_1 = m_\nu = 0$, we must have the minus sign
- For $\lambda_2 = M$ we have, plus sign

Eigenvalues Of Mass Matrix M

$\lambda_1 = m_\nu = 0$, (-) $\longrightarrow$ $$m_M^R m_M^L = m_D^2 \tag{7}$$

$\lambda_2 = M$, (+) $\longrightarrow$ $$m_M^R + m_M^L = M \tag{8}$$

We can calculate the eigenvector N (for λ_2) expressed in the ($\nu_L \;\; \nu_R^c)^T$ basis, from which we get the two equations:

$$(m_M^L - (m_M^R + m_M^L))\nu_c^L + m_D \nu_R = 0$$
$$m_D \nu_L^c + (m_v^R - (m_M^R + m_M^L))\nu_R = 0 \tag{9}$$

We get: $$\nu_L^c = \frac{m_D}{m_M^R}\nu_R \xrightarrow[\text{eigenvector}]{} N = \begin{bmatrix} \frac{m_D}{m_M^R}\nu_R \\ \nu_R \end{bmatrix} \tag{10}$$

Eigenvalues Of Mass Matrix M

If we include Hermitian conjugate, we can write true N in terms of the fields themselves, rather than as a two-component vector, expressed as:

$$N = (\nu_R + \nu_R^c) + \frac{m_D}{m_M^R}(\nu_L + \nu_L^c) \tag{11}$$

$$\nu = (\nu_L + \nu_L^c) - \frac{m_D}{m_M^R}(\nu_R + \nu_R^c) \tag{12}$$

If we now assume: $m_M^R \gg m_D$

ν_R can be thought of as composed almost entirely of N

ν_L is almost entirely composed of the weightless ν

Seesaw Mechanism

From

$$m_M^R m_M^L = m_D^2$$

we see that for a given value of m_D , a higher value for m_M^R means a lower the value for m_M^L, this is referred as "see-saw mechanism".

We can think of m_D as geometric mean between m_M^R and m_M^L.

Also, if

$$m_M^R \gg m_D$$

Holds, From

$$m_M^R + m_M^L = M \longrightarrow m_M^R \approx M$$

The mass hierarchy appears naturally:

$$M \approx m_M^R \gg m_D > m_M^L \approx 0 \tag{13}$$

Mass In Seesaw Mechanism And Predictions

- In the seesaw mechanism, it is hypothesized that the Dirac mass terms for the neutrinos are of a similar size to the masses of the other fermions, **O(1 GeV).** The Majorana mass M is then made sufficiently large that the lighter of the two physical neutrino states has a mass **mν ∼ 0.01 eV**. In this way, the masses of the lighter neutrino states can be made to be very small, even when the Dirac mass term is of the same order of magnitude as the other fermions. For this to work, the Majorana mass must be very large**, M ∼ 1011 GeV**.
- If a Majorana mass term exists, the seesaw mechanism predicts that for each of the three neutrino generations, there is a very light neutrino with a mass much smaller than the other Standard Model fermions and a very massive neutrino state $\boldsymbol{m_N} \approx$ **M**.
- The couplings of the light neutrinos to the weak charged-current are essentially the same as those of the Standard Model. And Since the massive neutrino state is almost entirely right-handed, it would not participate in the weak charged or neutral-currents.

Majorana Neutrinos vs Dirac Neutrinos

Dirac Neutrinos:

- have distinct particle and antiparticle states
- Does not conserve weak charge
- Does conserve lepton number

$$L_D = -m_D(\nu_R \nu_L + \nu_L \nu_R)$$

The first term destroys a RH particle and creates a LH one. Thus weak (chiral) charge is not conserved, as a LH neutrino has +1/2 weak charge, and a RH neutrino has zero weak charge.

Majorana Neutrinos:

- neutrinos are their own antiparticles
- Does not conserve weak charge
- Does not conserve lepton number
- Possibility of double beta decay, where a nucleus emits two electrons and no antineutrinos

$$L_M = -\frac{1}{2} m_M^L \left(\overline{\nu_L} \nu_L^c + \overline{\nu_L^c} \nu_L \right) - \frac{1}{2} m_M^R \left(\overline{\nu_R} \nu_R^c + \overline{\nu_R^c} \nu_R \right)$$

The first term in creates two LH neutrinos out of the vacuum and thus also does not conserve weak charge. But importantly, it doe not conserve lepton number. We started with zero neutrinos and ended up with two neutrinos.

მადლობა ყურადღებისათვის

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