

# Georgi-Glashow Model

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- **Howard Georgi** (Howard Mason Georgi III, Jan 6, 1947) - American theoretical physicist. Now - Professor at Harvard University.
- **Sheldon Lee Glashow** (Dec 5, 1932 ) - American theoretical physicist. Now - Professor at Harvard University & Boston University.
- **Georgi-Glashow Model:**  
The first Grand Unification Theory  
Standard Model  $\rightarrow$   $SU(5)$  Lie group



$$\psi_R \sim \psi_L^* \equiv \psi_L^c$$

r,g,y - colors

- Only first generation of fermions (15 in total, no right neutrino)

$$u^r, u^g, u^y, u^{cr}, u^{cg}, u^{cy}, d^r, d^g, d^y, d^{cr}, d^{cg}, d^{cy}, e^-, e^+, \nu_e$$

Strong interaction  $SU(3)$ :  $3^2 - 1 = 8$  gauge bosons and 8 generators  
(Gell-Mann matrices)

Triplet representation  $u \sim 3, u^c \sim 3^*, d \sim 3, d^c \sim 3^*$

Singlet representation  $e \sim 1, e^c \sim 1, \nu_e \sim 1$

$$3 \oplus 3^* \oplus 3 \oplus 3^* \oplus 1 \oplus 1 \oplus 1 \text{ of } SU(3)$$

- "Weak interaction"  $SU(2)$ :

$2^2 - 1 = 3$  gauge bosons and 3 generators (Pauli matrices)

For  $W^\pm$  bosons (we save up third boson for later)

Doublet representation  $\sim \begin{pmatrix} u \\ d \end{pmatrix}, \sim \begin{pmatrix} e \\ \nu_e \end{pmatrix}$

Singlet representation  $u^c \sim 1, d^c \sim 1, e^c \sim 1$

$$SU(3) \otimes SU(2)$$

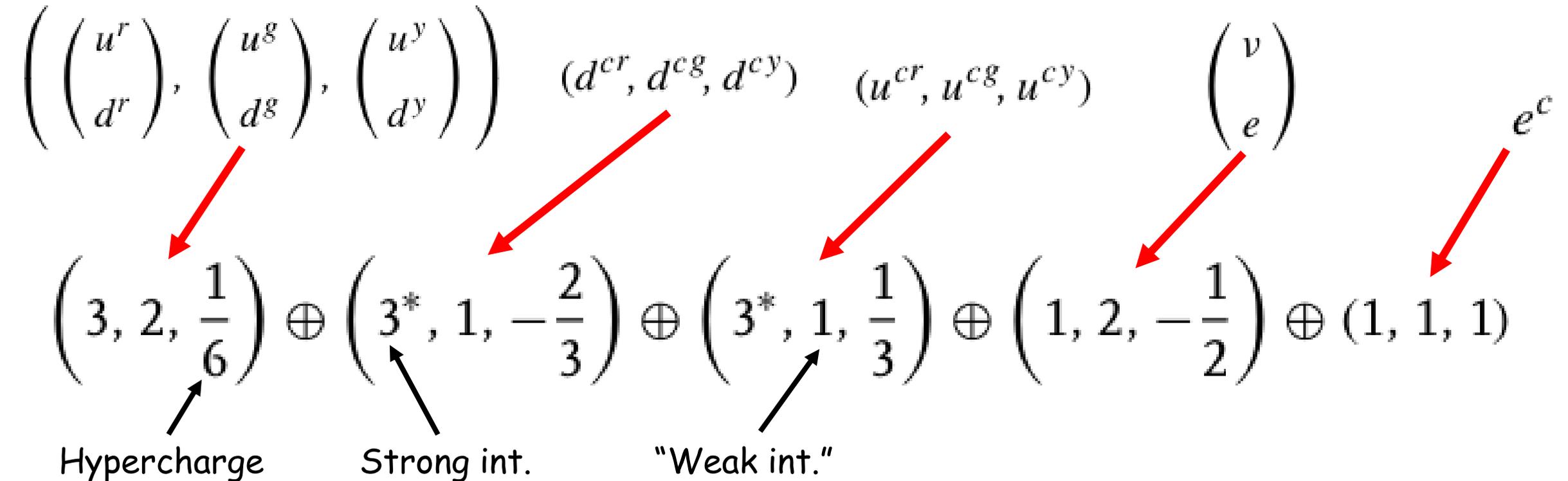
$$\left( \begin{pmatrix} u^r \\ d^r \end{pmatrix}, \begin{pmatrix} u^g \\ d^g \end{pmatrix}, \begin{pmatrix} u^y \\ d^y \end{pmatrix} \right), \begin{pmatrix} \nu \\ e \end{pmatrix}, (u^{cr}, u^{cg}, u^{cy}), (d^{cr}, d^{cg}, d^{cy}), e^c$$

• "Electromagnetic interaction" U(1)

$Q$  – electric charge  
 $T^3$  – 3<sup>rd</sup> projection of isospin  
 $Y$  – hypercharge

$$Q = T^3 + \frac{1}{2}Y$$

Only 1 generator –  $\frac{1}{2}Y \equiv$  average electric charge  $Q$  in the representation



- Linear combinations of third gauge boson (of  $SU(2)$ ) and gauge boson of  $U(1)$   $\rightarrow$   
 $\rightarrow Z^0$  boson and  $\gamma$

## Grand Unification, $SU(5)$

Sum of all hypercharges  $\equiv 0 \rightarrow$  generator of  $U(1)$  can be generator of  $SU(N)$  or  $SO(N)$  group as well!

We use:  $\frac{1}{2}Y = \begin{pmatrix} -\frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$

We have:  $5^2 - 1 = 24$  gauge bosons and 24 generators.

5x5 hermitian traceless matrices acting on five objects  $\psi^\mu$  (first representation)

Lets separate:

$$\psi^1, \psi^2, \psi^3$$

$$\psi^4, \psi^5$$

$$\psi^\alpha$$

$$\psi^i$$

Now 8 matrices  $\begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix}$  act on  $\psi^\alpha$ , where  $A$  is 3x3. ----- SU(3)

Similarly 3 matrices  $\begin{pmatrix} 0 & 0 \\ 0 & B \end{pmatrix}$  act on  $\psi^i$ , where  $B$  is 2x2. ----- SU(2)

$$5 \rightarrow \left( 3, 1, -\frac{1}{3} \right) \oplus \left( 1, 2, \frac{1}{2} \right) \quad \text{Taking conjugate:} \quad 5^* \rightarrow \left( 3^*, 1, \frac{1}{3} \right) \oplus \left( 1, 2, -\frac{1}{2} \right)$$

We know that

$$5 \rightarrow (3, 1, -\frac{1}{3}) \oplus (1, 2, \frac{1}{2})$$

Second representation - tensor representation  $\psi^{\mu\nu}$   
 $\psi^{\mu\nu}$  is antisymmetric with 10 independent elements

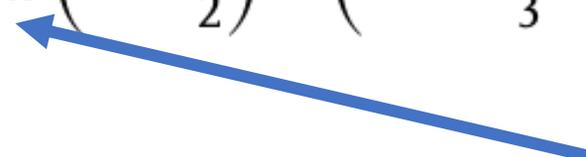
$$\left(3, 1, -\frac{1}{3}\right) \otimes_A \left(3, 1, -\frac{1}{3}\right) = \left(3^*, 1, -\frac{2}{3}\right)$$

$$\left(3, 1, -\frac{1}{3}\right) \otimes_A \left(1, 2, \frac{1}{2}\right) = \left(3, 2, -\frac{1}{3} + \frac{1}{2}\right) = \left(3, 2, \frac{1}{6}\right)$$

and

$$\left(1, 2, \frac{1}{2}\right) \otimes_A \left(1, 2, \frac{1}{2}\right) = (1, 1, 1)$$

antisymmetric product



$$10 \rightarrow \left(3, 2, \frac{1}{6}\right) \oplus \left(3^*, 1, -\frac{2}{3}\right) \oplus (1, 1, 1)$$

# Adding everything up

$$SU(3) \otimes SU(2) \otimes U(1)$$

Was combined into 5\* and 10 representation of SU(5)!

$$\psi_\mu = \begin{pmatrix} \psi_\alpha \\ \psi_i \end{pmatrix} = \begin{pmatrix} d^c \\ d^c \\ d^c \\ \nu \\ e \end{pmatrix}$$

$$\psi^{\mu\nu} = \{\psi^{\alpha\beta}, \psi^{\alpha i}, \psi^{ij}\} = \begin{pmatrix} 0 & u^c & -u^c & d & u \\ -u^c & 0 & u^c & d & u \\ u^c & -u^c & 0 & d & u \\ -d & -d & -d & 0 & e^c \\ -u & -u & -u & -e^c & 0 \end{pmatrix}$$

24 - 12 = 12 bosons left.

They transform  $\psi_\mu$  to  $\psi^{\mu\nu}$

SU(5):

- Preserves B-L number (baryon - lepton number), but not B & L alone;
- Predicts proton decay;
- Is a subgroup of more unifying group SO(10)

# Thanks for listening!



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