Georgi-Glashow Model

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- **Howard Georgi** (Howard Mason Georgi III, Jan 6, 1947) – American theoretical physicist. Now – Professor at Harvard University.

- **Sheldon Lee Glashow** (Dec 5, 1932) – American theoretical physicist. Now – Professor at Harvard University & Boston University.

- **Georgi-Glashow Model**:

The first Grand Unification Theory

Standard Model -> SU(5) Lie group

$\psi_R \sim \psi_L^* \equiv \psi_L^c$

r,g,y – colors

- Only first generation of fermions (15 in total, no right neutrino)

$$u^r,\ u^g,\ u^y,\ u^{cr},\ u^{cg},\ u^{cy},\ d^r,\ d^g,\ d^y,\ d^{cr},\ d^{cg},\ d^{cy},\ e^-,\ e^+,\ \nu_e$$

Strong interaction SU(3): $3^2 - 1 = 8$ gauge bosons and 8 generators (Gell-Mann matrices)

Triplet representation $u \sim 3,\ u^c \sim 3^*,\ d \sim 3,\ d^c \sim 3^*$

Singlet representation $\mathrm{e} \sim 1,\ e^c \sim 1,\ \nu_e \sim 1$

$$3 \oplus 3^* \oplus 3 \oplus 3^* \oplus 1 \oplus 1 \oplus 1 \text{ of } SU(3)$$

- “Weak interaction” SU(2):

$2^2 - 1 = 3$ gauge bosons and 3 generators (Pauli matrices)

For $W^{\pm}$ bosons (we save up third boson for later)

Doublet representation $\sim \binom{u}{d}, \sim \binom{e}{\nu_e}$

Singlet representation $u^c \sim 1,\ d^c \sim 1,\ e^c \sim 1$

$$SU(3) \otimes SU(2)$$

$$\left(\begin{pmatrix} u^r \\ d^r \end{pmatrix}, \begin{pmatrix} u^g \\ d^g \end{pmatrix}, \begin{pmatrix} u^y \\ d^y \end{pmatrix} \right), \begin{pmatrix} \nu \\ e \end{pmatrix}, \ (u^{cr}, u^{cg}, u^{cy}), \ (d^{cr}, d^{cg}, d^{cy}), \ e^c$$

- "Electromagnetic interaction" U(1)

Q – electric charge
T^3 – 3^{rd} projection of isospin
Y – hypercharge

$$Q = T^3 + \frac{1}{2}Y$$

Only 1 generator $-\ \frac{1}{2}Y$ $\equiv$ average electric charge Q in the representation

$$\left(\begin{pmatrix} u^r \\ d^r \end{pmatrix}, \begin{pmatrix} u^g \\ d^g \end{pmatrix}, \begin{pmatrix} u^y \\ d^y \end{pmatrix} \right) \quad (d^{cr}, d^{cg}, d^{cy}) \quad (u^{cr}, u^{cg}, u^{cy}) \quad \begin{pmatrix} \nu \\ e \end{pmatrix} \quad e^c$$

$$\left(3, 2, \frac{1}{6}\right) \oplus \left(3^*, 1, -\frac{2}{3}\right) \oplus \left(3^*, 1, \frac{1}{3}\right) \oplus \left(1, 2, -\frac{1}{2}\right) \oplus (1, 1, 1)$$

Hypercharge Strong int. "Weak int."

- Linear combinations of

third gauge boson (of SU(2)) and gauge boson of U(1) ->

-> Z^0 **boson and** γ

Grand Unification, SU(5)

Sum of all hypercharges $\equiv 0$ -> generator of U(1) can be generator of SU(N) or SO(N) group as well!

We use:

$$\frac{1}{2}Y = \begin{pmatrix} -\frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

We have: $5^2 - 1 = 24$ gauge bosons and 24 generators.
5x5 hermitian traceless matrices acting on five objects ψ^μ (first representation)
Lets separate:

$$\psi^1, \psi^2, \psi^3 \qquad\qquad \psi^4, \psi^5$$

$$\psi^\alpha \qquad\qquad\qquad \psi^i$$

Now 8 matrices $\begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix}$ act on ψ^α, where A is 3x3. ----- SU(3)

Similarly 3 matrices $\begin{pmatrix} 0 & 0 \\ 0 & B \end{pmatrix}$ act on ψ^α, where B is 2x2. ----- SU(2)

$5 \to \left(3, 1, -\frac{1}{3}\right) \oplus \left(1, 2, \frac{1}{2}\right)$ Taking conjugate: $5^* \to \left(3^*, 1, \frac{1}{3}\right) \oplus \left(1, 2, -\frac{1}{2}\right)$

We know that

$$5 \rightarrow (3, 1, -\tfrac{1}{3}) \oplus (1, 2, \tfrac{1}{2})$$

Second representation – tensor representation $\psi^{\mu\nu}$

$\psi^{\mu\nu}$ is antisymmetric with 10 independent elements

$$\left(3, 1, -\frac{1}{3}\right) \otimes_A \left(3, 1, -\frac{1}{3}\right) = \left(3^*, 1, -\frac{2}{3}\right)$$

$$\left(3, 1, -\frac{1}{3}\right) \otimes_A \left(1, 2, \frac{1}{2}\right) = \left(3, 2, -\frac{1}{3} + \frac{1}{2}\right) = \left(3, 2, \frac{1}{6}\right)$$

antisymmetric product

and

$$\left(1, 2, \frac{1}{2}\right) \otimes_A \left(1, 2, \frac{1}{2}\right) = (1, 1, 1)$$

$$10 \rightarrow \left(3, 2, \frac{1}{6}\right) \oplus \left(3^*, 1, -\frac{2}{3}\right) \oplus (1, 1, 1)$$

Adding everything up

$SU(3) \otimes SU(2) \otimes U(1)$ Was combined into 5* and 10 representation of SU(5)!

$$\psi_\mu = \begin{pmatrix} \psi_\alpha \\ \psi_i \end{pmatrix} = \begin{pmatrix} d^c \\ d^c \\ d^c \\ \nu \\ e \end{pmatrix} \qquad \psi^{\mu\nu} = \{\psi^{\alpha\beta}, \psi^{\alpha i}, \psi^{ij}\} = \begin{pmatrix} 0 & u^c & -u^c & d & u \\ -u^c & 0 & u^c & d & u \\ u^c & -u^c & 0 & d & u \\ -d & -d & -d & 0 & e^c \\ -u & -u & -u & -e^c & 0 \end{pmatrix}$$

24 – 12 = 12 bosons left.
They transform ψ_μ to $\psi^{\mu\nu}$

SU(5):

- Preserves B-L number (baryon – lepton number), but not B & L alone;
- Predicts proton decay;
- Is a subgroup of more unifying group SO(10)

Thanks for listening!

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