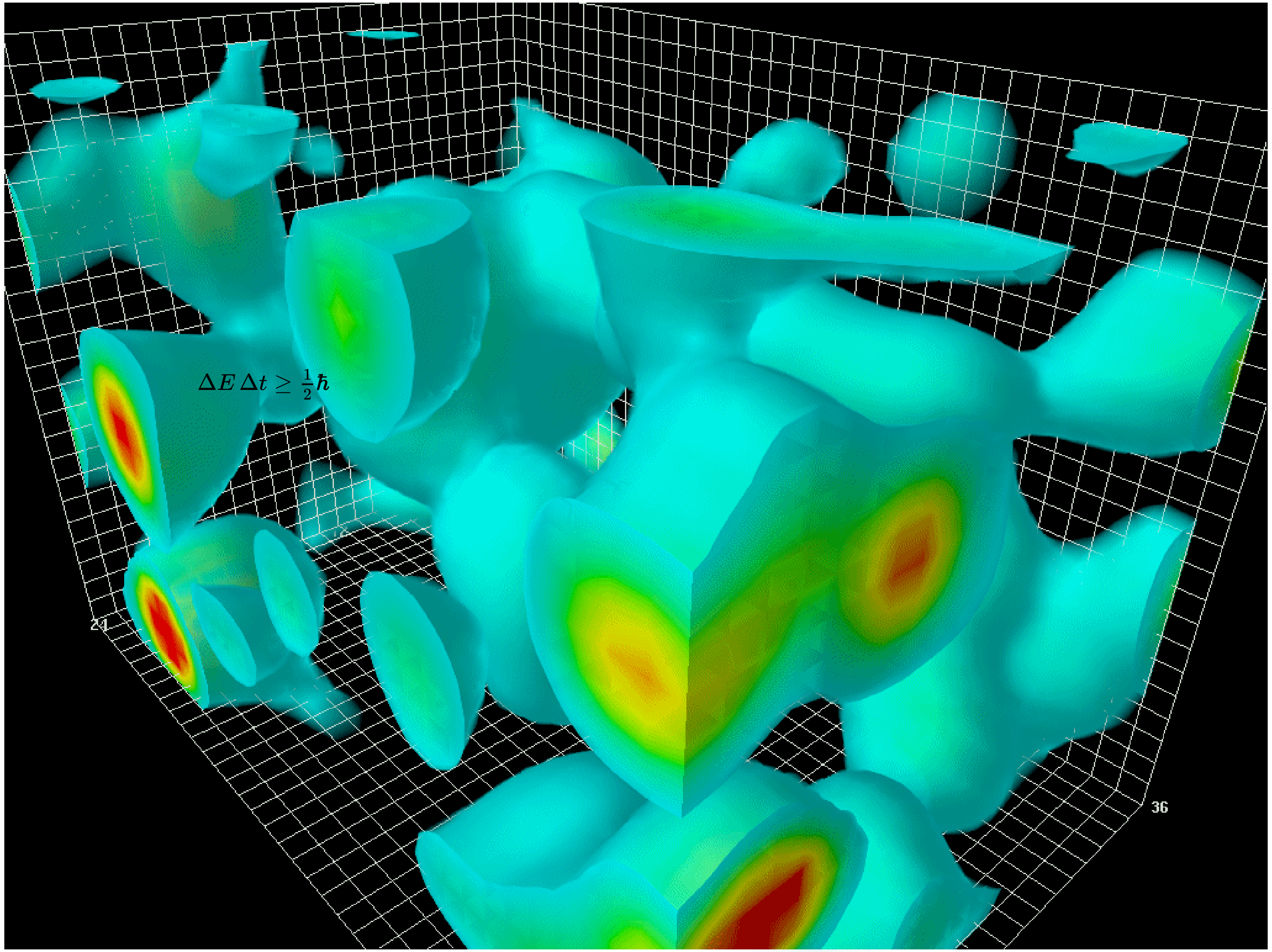
კაზიმირის ეფექტი

ლევან ლოლაძე

ვაკუუმის ფლუქტუაციები

$$\Delta t \Delta E \geq \frac{\hbar}{2}$$



ულტრა იისფერი და ინფრაწითელი განშლადობა

$$H = \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\,\omega_{\vec{p}}\,\left[a_{\vec{p}}a_{\vec{p}}^\dagger + a_{\vec{p}}^\dagger a_{\vec{p}}\right]$$
$$= \int \frac{d^3p}{(2\pi)^3}\,\omega_{\vec{p}}\,\left[a_{\vec{p}}^\dagger a_{\vec{p}} + \tfrac{1}{2}(2\pi)^3\,\delta^{(3)}(0)\right]$$

$$H\,|0\rangle \equiv E_0\,|0\rangle = \left[\int d^3p\,\frac{1}{2}\,\omega_{\vec{p}}\,\delta^{(3)}(0)\right]|\,0\rangle = \infty\,|0\rangle$$

$$\mathcal{E}_0 = \frac{E_0}{V} = \int \frac{d^3p}{(2\pi)^3}\,\frac{1}{2}\,\omega_{\vec{p}}$$

$$H = \int \frac{d^3p}{(2\pi)^3}\,\omega_{\vec{p}}\,a_{\vec{p}}^\dagger a_{\vec{p}}$$

კაზემირის ეფექტი

$$\phi(\vec{x}) = \phi(\vec{x} + L\vec{n})$$

$$\vec{p} = \left(\frac{n\pi}{d}, p_y, p_z\right) \qquad n \in \mathbf{Z}^+$$

$$\frac{E(d)}{A} = \sum_{n=1}^{\infty} \int \frac{dp_y dp_z}{(2\pi)^2} \frac{1}{2} \sqrt{\left(\frac{n\pi}{d}\right)^2 + p_y^2 + p_z^2}$$

$$E = E(d) + E(L-d)$$

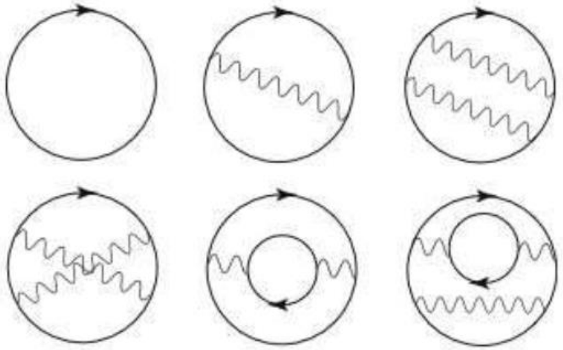
$$\frac{E(d)}{A} = \sum_{n=1}^{\infty} \int \frac{dp_y dp_z}{(2\pi)^2} \frac{1}{2} \left(\sqrt{\left(\frac{n\pi}{d}\right)^2 + p_y^2 + p_z^2}\right) \; e^{-a\sqrt{\left(\frac{n\pi}{d}\right)^2 + p_y^2 + p_z^2}}$$

$$E_{1+1}(d) = \frac{\pi}{2d} \sum_{n=1}^{\infty} n$$

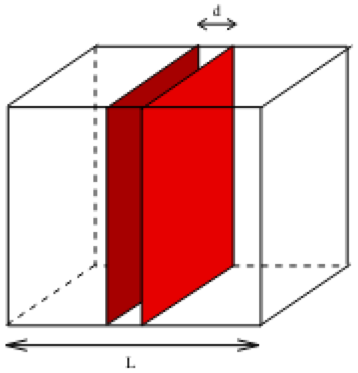
$$E_{1+1}(d) \rightarrow \frac{\pi}{2d} \sum_{n=1}^{\infty} n\, e^{-an\pi/d}$$

$$E_{1+1} = E_{1+1}(d) + E_{1+1}(L-d) = \frac{L}{2\pi a^2} - \frac{\pi}{24}\left(\frac{1}{d} + \frac{1}{L-d}\right) + \mathcal{O}(a^2)$$

$$\frac{1}{A}\frac{\partial E}{\partial d} = \frac{\pi^2}{480 d^4}$$



1: QED graphs contributing to the zero point energy



კაზემირ-პოლდერის ძალა

$$\Delta E = -\frac{23\hbar c}{4\pi R^7} a_1 a_2 \, .$$

$$F(a) = -\frac{\pi^2}{240} \frac{\hbar c}{a^4} S$$

$$F(a) = -\frac{\pi^2}{240} \frac{\hbar c}{a^4} \frac{(\epsilon_0 - 1)^2}{(\epsilon_0 + 1)^2} \varphi(\epsilon_0) \; S \, ,$$

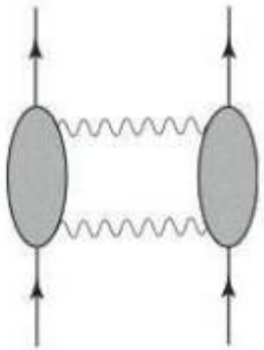
$$\mathcal{E} = \frac{\hbar}{2\pi} \mathrm{Im} \int d\omega\omega \, \mathrm{Tr} \int d^3x \left[\mathcal{G}(x,x,\omega + i\varepsilon) - \mathcal{G}_0(x,x,\omega + i\varepsilon) \right]$$

$$\frac{1}{\pi} \mathrm{Im} \int \left[\mathcal{G}(x,x,\omega + i\varepsilon) - \mathcal{G}_0(x,x,\omega + i\varepsilon) \right] = \frac{d\Delta N}{d\omega}$$

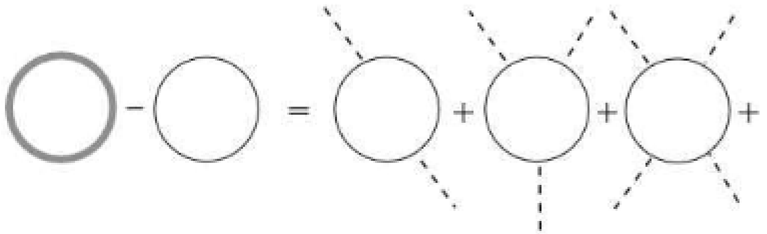
$$\mathcal{L}_{\mathrm{int}} = \frac{1}{2} g \sigma(x) \phi^2(x)$$

$$F(a,g,m) = -\frac{g^2}{\pi} \int_m^\infty \frac{t^2 dt}{\sqrt{t^2 - m^2}} \frac{e^{-2at}}{4t^2 + 4gt + g^2(1 - e^{-2at})}$$

$$\lim_{g \to \infty} F(a,g,m) = -\int_m^\infty \frac{dt}{\pi} \frac{t^2}{\sqrt{t^2 - m^2}\,(e^{2ta} - 1)} \, ,$$



2: Feynman diagrams for the Casimir-Polder force



3: Diagrammatic expansion of the Casimir force: The thick (thin) line denotes the full (free) Greens function; the one-point function is omitted because it does not contribute to the force.

გამოყენებული ლიტერატურა

- David Tong: Lectures on Quantum Field Theory- https://www.damtp.cam.ac.uk/user/tong/qft.html
- New Developments in the Casimir Effect M. Bordag , U. Mohideen, V.M. Mostepanenko https://arxiv.org/pdf/quant-ph/0106045
- The Casimir Effect and the Quantum Vacuum R. L. Jaffe- https://arxiv.org/pdf/hep-th/0503158

გმადლობთ ყურადღებისთვის