Gravitoelectromagnetism

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Introduction to Gravitoelectromagnetism

Gravitoelectromagnetism equations

$$\nabla \cdot \boldsymbol{E}_g = -4\pi\sqrt{G}\rho$$

$$\nabla \cdot \boldsymbol{B}_g = 0$$

$$\nabla \times \boldsymbol{E}_g = -\frac{1}{c}\frac{\partial \boldsymbol{B}_g}{\partial t}$$

$$\nabla \times \boldsymbol{B}_g = -\frac{4\pi}{c}\sqrt{G}\boldsymbol{J} + \frac{1}{c}\frac{\partial \boldsymbol{E}_g}{\partial t}$$

$$\boldsymbol{f}_g = \sqrt{G}\rho\left(\boldsymbol{E}_g + \frac{4\mathbf{v}}{\mathrm{c}} \times \boldsymbol{H}\right)$$

Maxwell's equations

$$\nabla \cdot \boldsymbol{E} = 4\pi\rho$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{J} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t}$$

$$\boldsymbol{f}_g = \rho\left(\boldsymbol{E}_g + \frac{1}{\mathrm{c}}\mathbf{v} \times \boldsymbol{H}\right)$$

- Intermediate theory of gravity
- Not nearly as empirically successful as general relativity but explains some of the effects Newton's gravity fails to explain.
- Can be derived from either as an extension of Newtonian gravity modeled off electromagnetism or as a weak-field slow-velocity approximation of general relativity

Derivation by extending newtons gravity [Charles T. Sebens]

$$\nabla \cdot \boldsymbol{E}_g = -4\pi\sqrt{G}\rho$$
$$\nabla \cdot \boldsymbol{B}_g = 0$$
$$\nabla \times \boldsymbol{E}_g = -\frac{1}{c}\frac{\partial \boldsymbol{B}_g}{\partial t}$$
$$\nabla \times \boldsymbol{B}_g = -\frac{4\pi}{c}\sqrt{G}\boldsymbol{J} + \frac{1}{c}\frac{\partial \boldsymbol{E}_g}{\partial t}$$
$$\boldsymbol{f}_g = \sqrt{G}\rho\left(\boldsymbol{E}_g + \frac{4\mathbf{v}}{c} \times \boldsymbol{H}\right)$$

- If the relevant velocities are sufficiently small, the laws of electromagnetism reduce to the laws of electrostatics: $\nabla \cdot \boldsymbol{E} = 4\pi\rho$, $\nabla \times \boldsymbol{E} = 0$ and $\boldsymbol{f} = \rho\boldsymbol{E}$
- If we replace $\boldsymbol{g}$ by $\boldsymbol{E}_g = \frac{1}{\sqrt{G}}\boldsymbol{g}$ in the laws of Newtonian gravity with $\nabla \cdot \boldsymbol{g} = -4\pi G\rho$, $\boldsymbol{f}_g = \rho\boldsymbol{g}$, with addition of $\nabla \times \boldsymbol{g} = 0$. the electrostatic equations resemble the laws on Newtonian gravity.
- We can extend this replacement procedure to full electromagnetism to get a theory of gravitoelectromagnetism in which moving mass produces a 'gravitomagnetic field' just as moving charge produces a magnetic field.[Charles T. Sebens]

Derivation by taking limit in GR [Charles T. Sebens]

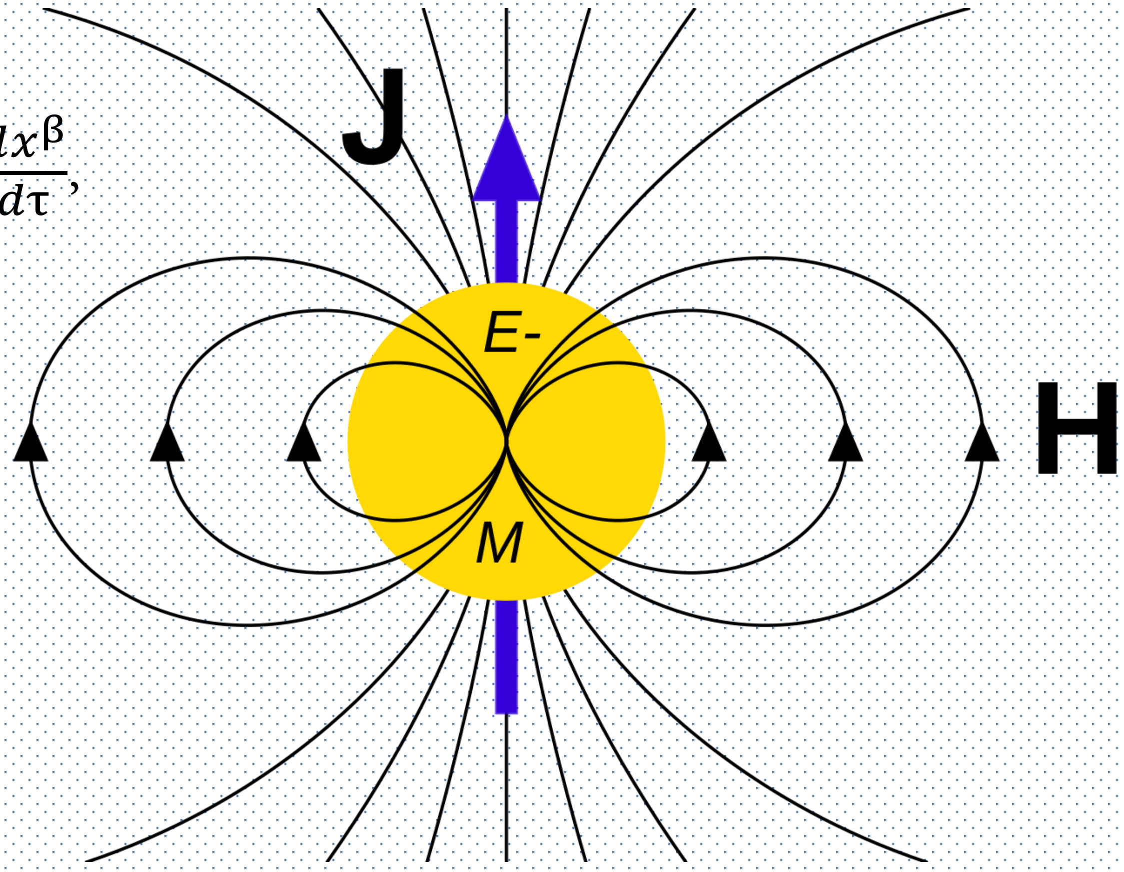
taking a linear approximation to GR $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu}$ is Minkowskian metric and $h_{\mu\nu}$ is a potential specifying the state of the field. In the case of reverse-trace of $h_{\mu\nu}$, $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^{\lambda}_{\lambda}$. Equation of GR takes the form:

$$\partial^{\lambda}\,\overline{h_{\mu\nu}} + \eta_{\mu\nu}\,\partial^{\rho}\,\partial^{\sigma}\,\overline{h_{\rho\sigma}} - \partial_{\nu}\,\partial^{\rho}\,\overline{h_{\rho\mu}} - \partial_{\mu}\,\partial^{\rho}\,\overline{h_{\rho\nu}} = -\frac{16\,\pi\,G}{c^4}\,T_{\mu\nu}$$

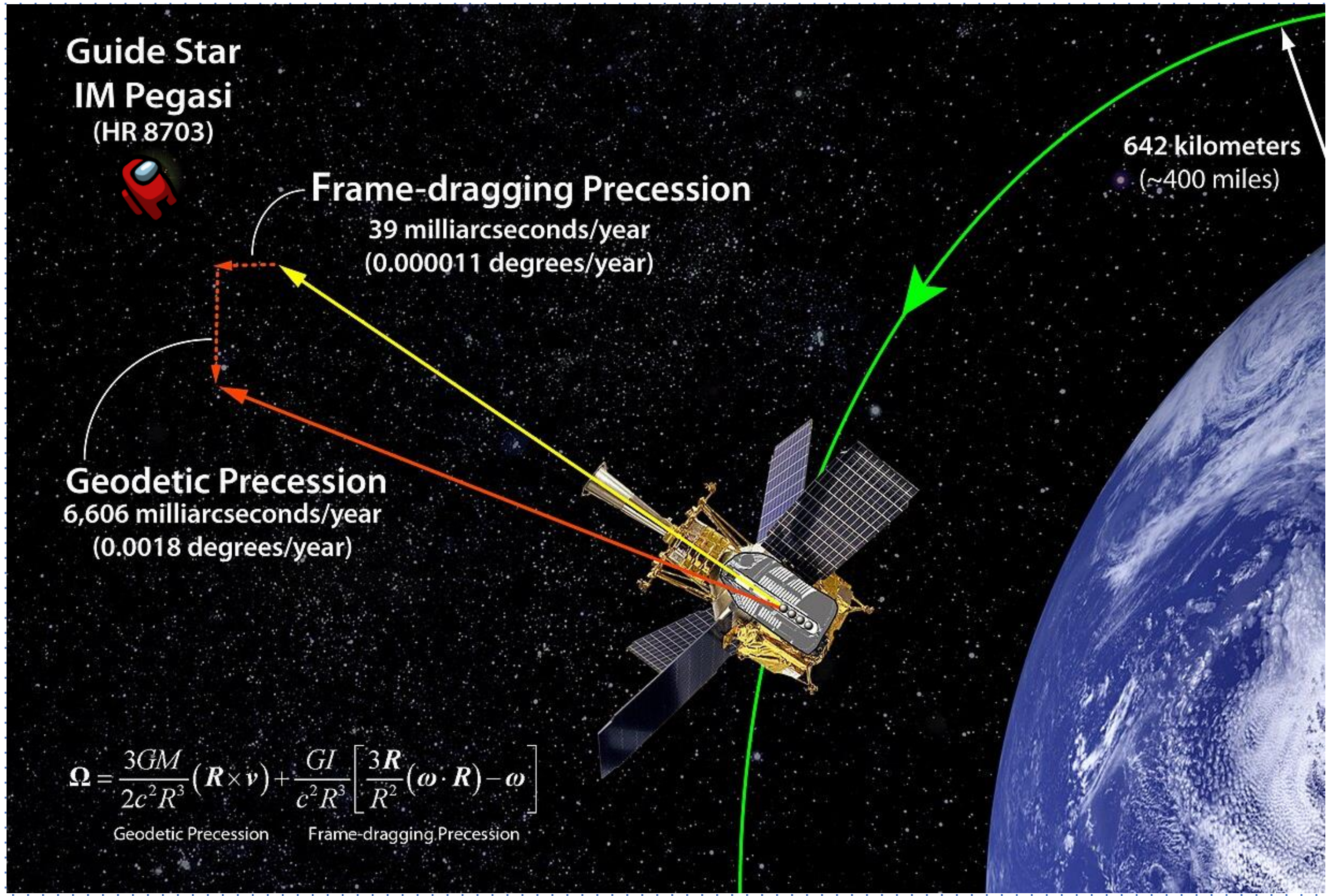
- We choose the Lorenz gauge for $h_{\mu\nu}$ in which$\partial_{\mu}\bar{h}^{\mu\nu} = 0$, the Einstein field equations simplify to $\partial^{\lambda}\partial_{\lambda}\overline{h_{\mu\nu}} = -\frac{16\pi G}{c^4}T_{\mu\nu}$.
- T_{ij} terms are sufficiently small and there is no source-free gravitational radiation, the $\overline{h_{ij}}$ terms are negligible and only $\overline{h_{0\mu}}$ needs to be considered.
- four-potential $A_{\mu} = \frac{c^2}{4\sqrt{G}}\overline{h_{0\mu}}$.
- The zeroth component yields $-\frac{1}{c}\frac{\partial A_0}{\partial t} = \nabla \cdot \vec{A}$ and 0μ component $\partial^{\lambda}\partial_{\lambda}A_{\mu} = \frac{4\pi\sqrt{G}}{c^2}T_{0\mu}$. This has the same form as the electromagnetic Maxwell equations for the four-potential in the Lorenz gauge.
- The two gravitoelectromagnetic equations with sources are just Lorenz gauge expressed in terms of the gravitoelectric and gravitomagnetic fields $\overrightarrow{E_g} = -\nabla\phi_g - \frac{1}{c}\frac{\partial \overrightarrow{A_g}}{\partial t}$ and $\overrightarrow{B_g} = \nabla \times \overrightarrow{A_g}$.
- The sourceless gravitoelectromagnetic Maxwell equations follow automatically from the way the fields are defined in terms of four-potential

The “Lorenz” force law [Charles T. Sebens]

- We start with geodesic equation for GR $\frac{d^2x^i}{d\tau^2} = -\Gamma^i_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$, and assume that that the velocity of our test particle is small enough that we can ignore terms of order β^2.
- $\vec{a} = \frac{d^2x^i}{dt^2} = -c^2\Gamma^i_{00} - c\Gamma^i_{j0}\frac{dx^j}{dt} - c\Gamma^i_{0j}\frac{dx^j}{dt}$
- Geodesic equation becomes $\vec{a} = \frac{c^2}{2}\partial_i\overline{h_{00}} + \frac{c}{4}\partial_i\overline{h_{00}} + 4\frac{\sqrt{G}}{c}\epsilon_{ijk}\frac{dx^j}{dt}B_{jk} = \sqrt{G}\left(\overrightarrow{E_g} + \frac{4}{c}\vec{v}\times\overrightarrow{B_g}\right)$

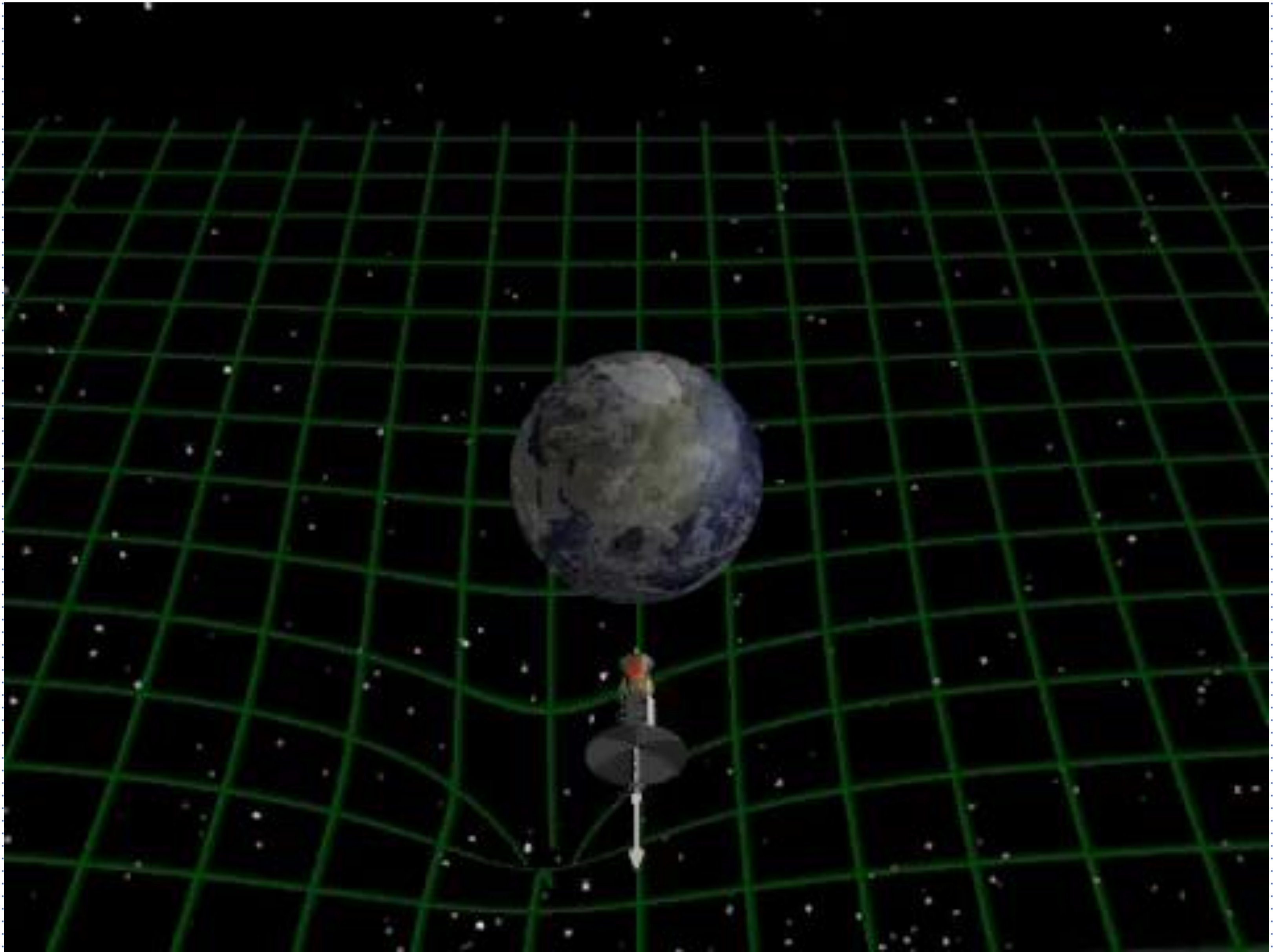


Effects explained by gravitoelectromagnetism

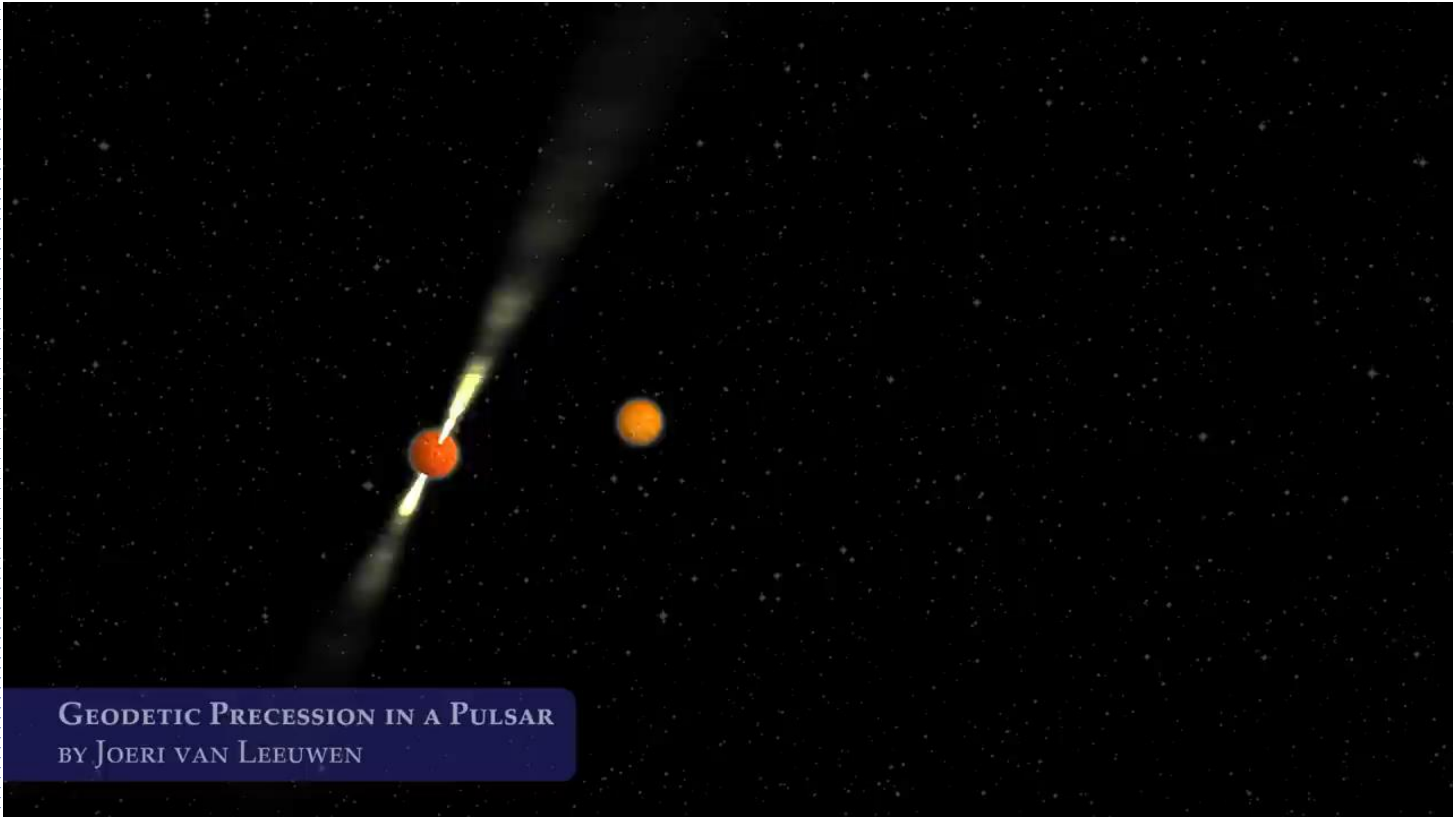


- Frame-Dragging / Lense-Thirring Precession.
- Geodetic Effect.
- Gravitomagnetic Clock Effect:
  $T = T_{\mathrm{Kep}} + T_{\mathrm{Gvm}} = T_{\mathrm{Kep}} \pm \frac{S}{Mc^2}$
  [Lorenzo Iorio et. al]
- Influence on High-Energy Astrophysical Phenomena
- Derivation of all the effects requires stating a problem in the rotating metric, like Schwarzschild or Kerr metric.

Frame dragging



Geodetic effect



Thanks

C.T. Sebens, *The mass of gravitational field*. **Br. J. Philos. Sci.**, **73** (2022) 211; doi:10.1093/bjps/axz002

Frame dragging : https://www.youtube.com/watch?v=ZgvjwEmKY6o

Geodetic effect : https://www.youtube.com/watch?v=IgihQG8t0kI

Gravitomagnetic clock effect: https://arxiv.org/pdf/2310.13118