Unruh Effect

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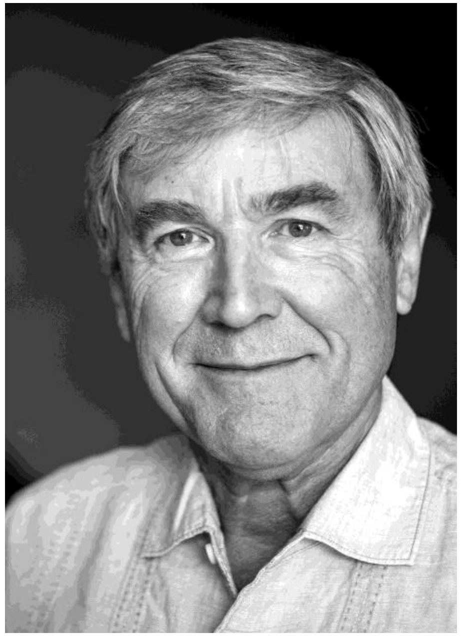
Plan of the talk

- What is the Unruh effect in the first place?
- How is it possible to outpace the light?
- Rindler space – SR review
- Massless Klein-Gordon in Rindler coordinates
- Bogoliubov transformation
- Derivation of the temperature
- Conclusions
- References

- # What is the Unruh effect?

Unruh effect (Fulling-Davies-Unruh effect) – An observer moving with uniform acceleration through the Minkowski vacuum observes a thermal spectrum of particles.

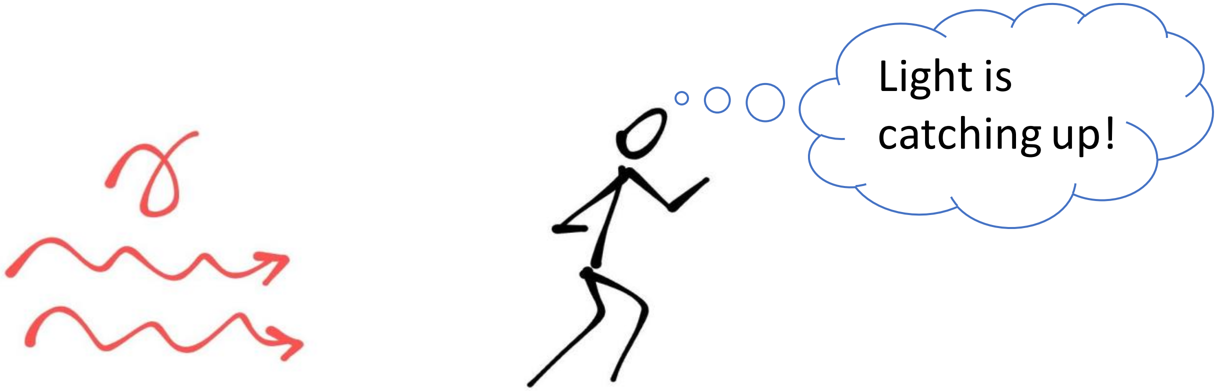


William Unruh – 1976
Stephen Fulling – 1973
Paul Davies – 1975

Unruh effect was first discovered in an attempt to understand the physics of Hawking radiation - thermal radiation in the presence of a black hole event horizon.

- # How is it possible to outpace the light?

Constant uniform acceleration is the answer.



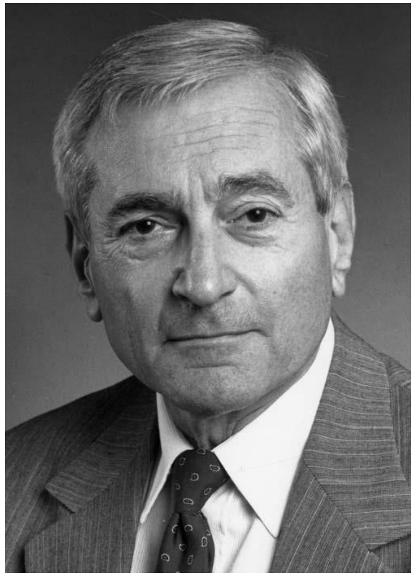
The act of acceleration cuts off observers causal access to a region of the universe – generates a type of event horizon. As a result, accelerating observers find themselves in a warm bath of particles, whose temperature is proportional to the acceleration.

Event horizon for accelerated observer – Rindler Horizon

Accelerated observer – Rindler observer

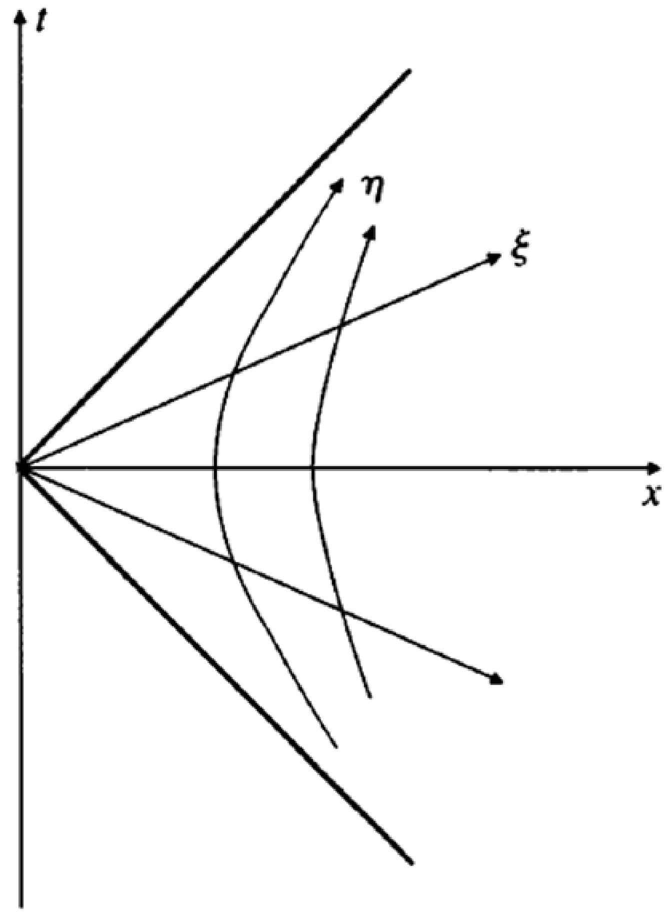
$$D \sim \frac{1}{a}$$

Distance of rindler horizon is inversely proportional to the acceleration



Wolfgang Rindler
(1924 – 2019)

• Rindler space – SR review



Rindler wedge

Let's consider 2D Minkowski spacetime as seen by a uniformly accelerating observer. Metric in inertial coordinates:

$$ds^2 = dt^2 - dx^2$$

An observer is moving at a uniform acceleration of magnitude a in the x-direction. Resulting trajectory $x^\mu(\tau)$ is given by:

$$t(\tau) = \frac{1}{a}\sinh a\tau\,; \quad x(\tau) = \frac{1}{a}\cosh a\tau$$

Let's verify that this path corresponds to constant acceleration:

$$a^\mu = \frac{d^2x^\mu}{d\tau^2} \;\rightarrow\; a^t = a\sinh a\tau\,;\;\; a^x = a\cosh a\tau$$

And the magnitude:

$$\sqrt{a^\mu a_\mu} = a$$

• Rindler space – SR review

We eventually want to look at quantum field from the perspective of the accelerating observer. We can choose new coordinates (η, ξ) that are naturally adapted to uniformly accelerated motion – Rindler coordinates.

Time coordinate $\boldsymbol{(\eta, \xi)}$ **Space coordinate**

$$t = \frac{1}{a} e^{a\xi} \sinh(a\eta), \qquad x = \frac{1}{a} e^{a\xi} \cosh(a\eta)$$

Metric in this coordinates takes the form:

$$g_{\mu\nu}{}^{R} = \begin{pmatrix} e^{2a\xi} & 0 \\ 0 & -e^{2a\xi} \end{pmatrix}$$

<u>Still Minkowski</u>

From the accelerated observers perspective, speed of light behind him is slower than c and in front of him it is faster than c. This is why he thinks he outpaces the light.

• Massless Klein-Gordon in Rindler coordinates

$(\partial^{\mu}\partial^{\nu}\eta_{\mu\nu} + m^2)\phi = 0$ inertial observer in Minkowski

$\eta_{\mu\nu} \rightarrow g_{\mu\nu}{}^{R}$

$(\partial^0\partial^0 e^{2a\xi} - \partial^1\partial^1 e^{2a\xi} + m^2)\phi = [e^{2a\xi}(\partial_\eta^2 - \partial_\xi^2) + m^2]\phi = 0$

$e^{2a\xi}$ is not scaling the mass term, which complicates the situation. For simplicity we consider massless field

$(\partial_\eta^2 - \partial_\xi^2)\phi = 0$ Klein-Gordon equation in rindler space for massless field.

Solution in cartesian coordinates:

$$\phi(\mathrm{t}, x) = \int \frac{dk}{2\pi\sqrt{2E_k}}(a(k)e^{-iEt}\,e^{ikx} + a^{+}(k)e^{iEt}e^{-ikx})$$

• Bogoliubov transformation

Creation and annihilation operators in Rindler space: $b(p), b^{+}(p)$.

We need correspondence:

$$b(p), b^{+}(p) \sim a(k), a^{+}(k)$$

Bogoliubov transformation:

$$b_p = \int_{-\infty}^{+\infty} dk \, (\alpha_{kp} a_k + \beta_{kp} a_k^{+})$$

Number operator:

$$\langle 0_c | N_R | 0_c \rangle = \int dk \int dp \, |\beta_{kp}|^2$$

The two observers will disagree on the number of particles in some general quantum state – Unruh effect.

• Bogoliubov transformation

Decomposing the scalar field in cartesian coordinates into positive and negative frequency components:

$$\phi(\mathrm{t},x) = \int \frac{dk}{2\pi\sqrt{2E_k}}\left(a(k)e^{-iEt}\,e^{ikx} + a^+(k)e^{iEt}e^{-ikx}\right)$$

$$= \int_0^\infty \frac{dk}{2\pi\sqrt{2k}}\left(a(-k)e^{ikx_+} + a^+(-k)e^{-ikx_+} + a(k)e^{ikx_-} + a^+(k)e^{-ikx_-}\right)$$

$x_+ = x + t;\quad x_- = x - t$ also for massless particles $E = p$

Let's do the same for the field in Rindler coordinates:

$$\phi(\mathrm{\eta},\xi) = \int_0^\infty \frac{dp}{2\pi p}\left(b(-p)e^{ik\tilde{x}_+} + b^+(-p)e^{-ik\tilde{x}_+} + b(p)e^{ik\tilde{x}_-} + b^+(p)e^{-ik\tilde{x}_-}\right)$$

$\tilde{x}_+ = \xi + \eta;\quad \tilde{x}_- = \xi - \eta$

Our goal is to express b operators in terms of a operators.

- # Derivation of the temperature

We declare that scalar field is the same, two observers agree on the actual field, so we write:

$$\phi(\mathrm{t}, x) = \phi(\eta, \xi)$$

This would imply relation between operators and we will see that they will not be the same – this is what gives rise to the Unruh effect. This equality gives:

$$\int_0^\infty \frac{dk}{2\pi\sqrt{2k}}\left(a(-k)e^{ikx_+} + a^+(-k)e^{-ikx_+}\right) = \int_0^\infty \frac{dp}{2\pi p}\left(b(-p)e^{ik\tilde{x}_+} + b^+(-p)e^{-ik\tilde{x}_+}\right)$$

$$\int_0^\infty \frac{dk}{2\pi\sqrt{2k}}\left(a(k)e^{ikx_-} + a^+(k)e^{-ikx_-}\right) = \int_0^\infty \frac{dp}{2\pi p}\left(b(p)e^{ik\tilde{x}_-} + b^+(p)e^{-ik\tilde{x}_-}\right)$$

To get operators in Rindler space, we must use Fourier transformation as well.

• Derivation of the temperature

After the Fourier transformation we get:

$$b(p) = \int_{-\infty}^{+\infty} d\tilde{x}_-\, e^{-ip\tilde{x}_-} \int_0^{\infty} \frac{dk}{2\pi} \sqrt{k/p}\left(a(k)e^{ikx_-} + a^{+}(k)e^{-ikx_-}\right)$$

Defining:

$$f(k,p) = \int_{-\infty}^{+\infty} \frac{d\tilde{x}_-}{2\pi} e^{ikx_-} e^{-ip\tilde{x}_-}$$

$$f(-k,p) = \int_{-\infty}^{+\infty} \frac{d\tilde{x}_-}{2\pi} e^{-ikx_-} e^{-ip\tilde{x}_-}$$

For the operator we get the expression:

$$b(k) = \int_0^{\infty} dk \sqrt{\frac{k}{p}} \left(a(k) f(k,p) + a^{+}(k) f(-k,p)\right)$$

We compare this to the Bogoliubov transformation and define Bogoliubov coefficients

- # Derivation of the temperature

$$\alpha_{kp} = \sqrt{\frac{k}{p}} f(k,p); \qquad \beta_{kp} = \sqrt{\frac{k}{p}} f(-k,p)$$

$$\langle 0_c|N_R|0_c\rangle = \int dk \int dp \left|\beta_{kp}\right|^2 = \int dk \int dp \left(\frac{k}{p}\right) |f(-k,p)|^2$$

This is the number of particles accelerating observer sees. Using bogoliubov coefficients and the properties:

$$[b(p), b^+(p')] = 2\pi\delta(p - p')$$

$$f(k,p) = f(-k,p)e^{\frac{p\pi}{a}}$$

We get:

$$\int dk \left(\frac{k}{p}\right) |f(-k,p)|^2 \left(e^{\frac{(p+p')\pi}{a}} - 1\right) = \delta(p - p')$$

- # Derivation of the temperature

$$\int dk\left(\frac{k}{p}\right)|f(-k,p)|^2=\frac{\delta(p\ -p')}{(e^{\frac{(p+p')\pi}{a}}-1)}$$

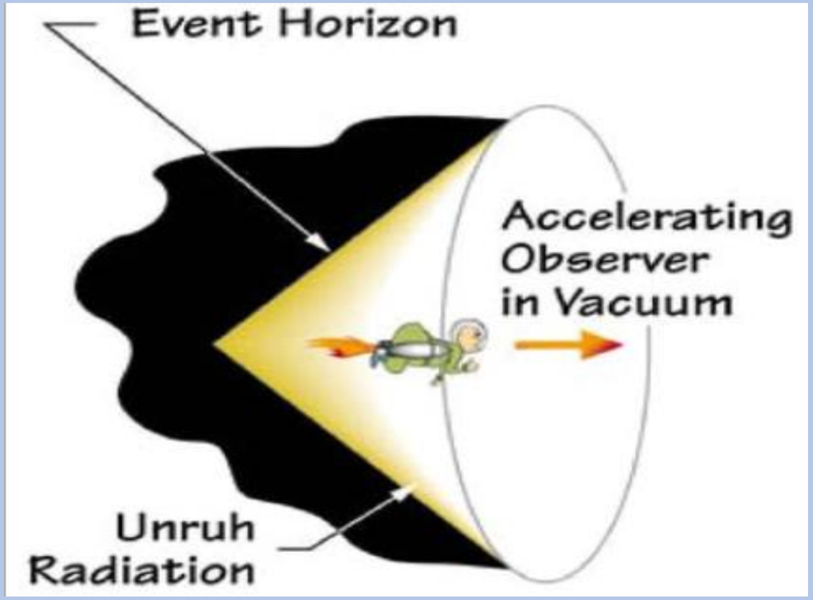
If $p=p'$ rhs is divergent, and since infinite amount of particles makes no sense we change it with a unit spatial volume:

$$\langle 0_c|N_R|0_c\rangle=\int_0^{\infty}dp\,\frac{1}{(e^{\frac{2p\pi}{a}}-1)}$$

According to Bose-Einstein distribution:

$$\frac{1}{(e^{\frac{E}{T}}-1)}=\frac{1}{(e^{\frac{2p\pi}{a}}-1)}\qquad\Rightarrow\qquad\boxed{\mathrm{T}=\frac{a}{2\pi}}$$

Conclusions



- Accelerated observer sees particle radiation which looks like black body radiation. Temperature can be calculated in QFT.
- The Unruh effect shows how two different sets of observers – inertial and rindler will describe the same state in very different terms. Vacuum appears warm with a $T \sim a$.
- Existence of particles – observer dependent?
- This effect is analogous to Hawking radiation near black holes, where an observer at a distance from the black hole perceives radiation that is not apparent to a freely falling observer crossing the event horizon

References:

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Thank You