Mathematics and Physics of Cosmic Strings

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Overview

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- Topological defects are - Domain Walls, Strings and Monopoles.

Phase Transitions and Topological Defects

Lets consider the following potential with $U(1)$ symmetry

$$V_T(\phi) = AT^2|\phi|^2 + \frac{\lambda}{2}\left(|\phi|^2 - \eta^2\right)^2$$

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- Simply connected but when can't shirnk a sphere to a point gives

Example of Domain Walls

- Lagrangian for the real field:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \frac{1}{2}\lambda\left(\phi^2 - \eta^2\right)^2$$

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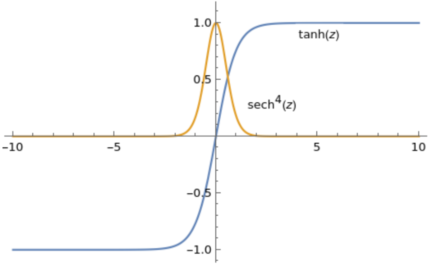
- Energy-Momentum Tensor:

$$T_\mu^\nu = \lambda\eta^4 \mathrm{sech}^4(\eta\sqrt{\lambda}z)$$

$$\sigma = \int T_0^0 dz = \frac{4}{3}\sqrt{\lambda}\eta^3$$

- 

Cosmic Strings

- Consider a lagrangian for the complex scalar field

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- Magnetic flux and Energy-Momentum Tensor

$$\int \boldsymbol{B} d\boldsymbol{S} = \frac{2\pi n}{g}$$

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- graph

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$$T^{\mu\nu}(\boldsymbol{r}, t) = \mu \int d\xi (\partial_t x^{\mu} \partial_t x^{\nu} - \partial_x x^{\mu} \partial_x x^{\nu}) \delta(\boldsymbol{r} - \boldsymbol{x}(\xi, t))$$

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- The angular momentum is

$$\boldsymbol{J} = \mu \int d\xi \, [\boldsymbol{x} \times \partial_t \boldsymbol{x}]$$

Bibliography

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Thank you for the Attention