Department of Physics, Faculty of Exact & Natural Sciences, Javakhishvili Tbilisi State University

Binary systems

Kavtaradze Mariami

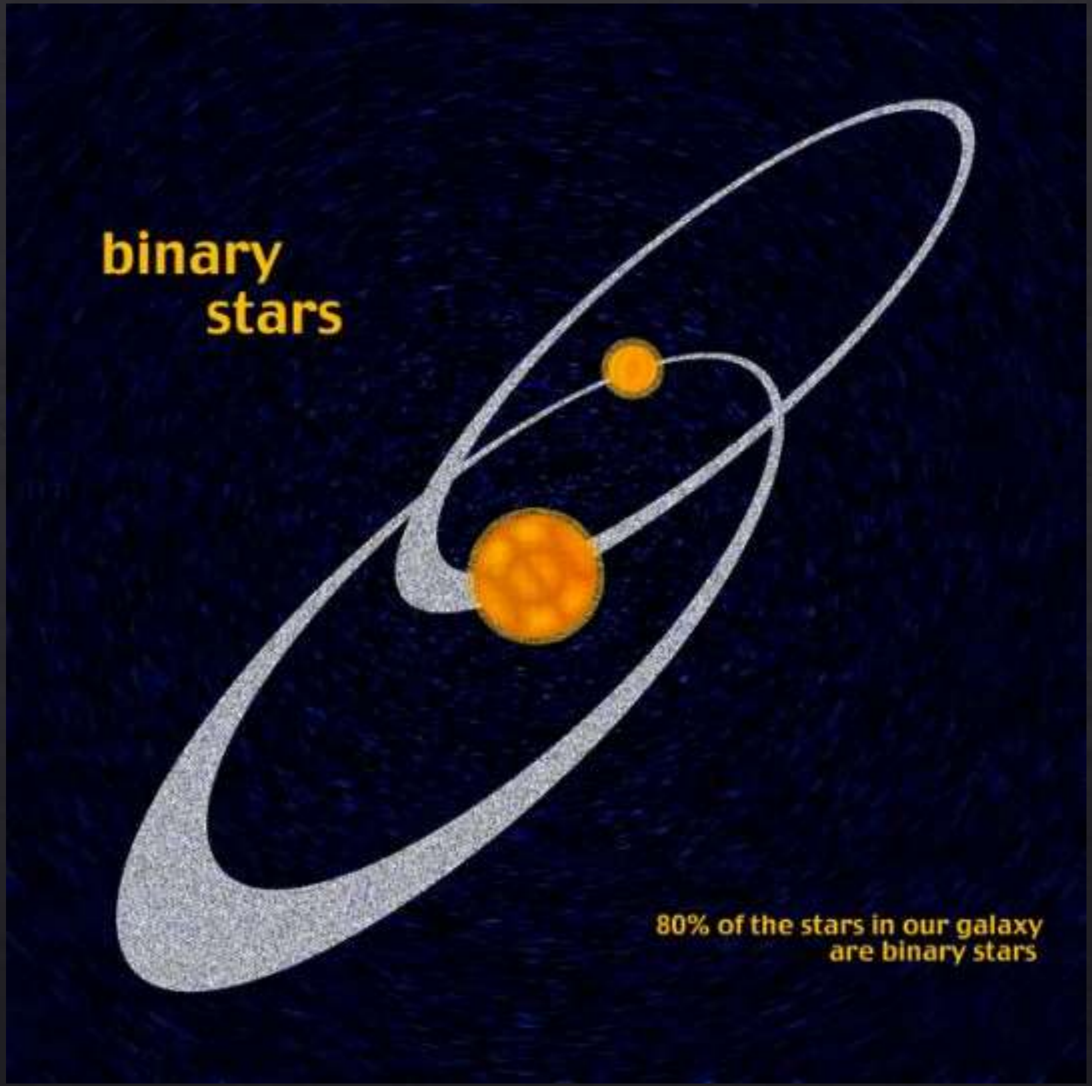
preposition

- What is Binary system
- Types of Binary system
- Accretion
- Evolution of Binary system
- Gravitational radiation

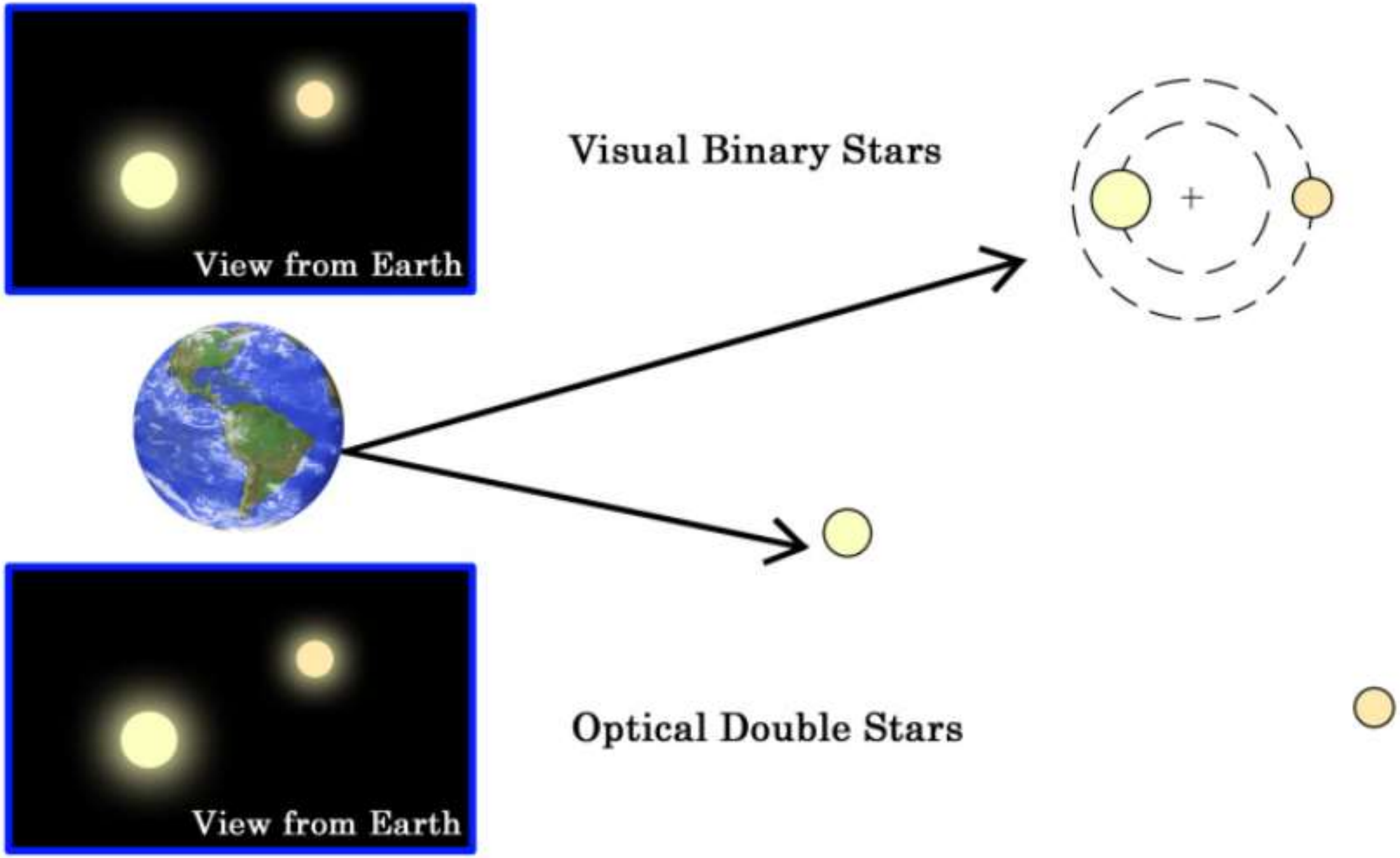
What is Binary system

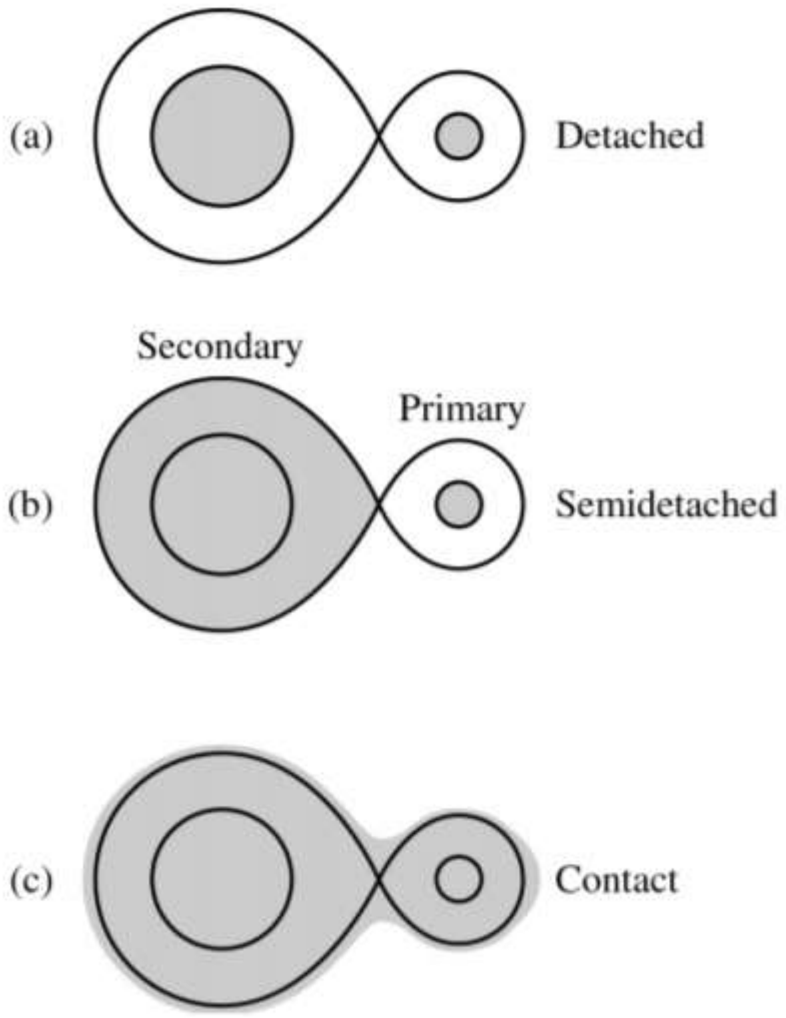
A **binary system** is a system of two astronomical bodies which are close enough that their gravitational attraction causes them to orbit each other around a barycenter. Research over the last two centuries suggests that half or more of visible stars are part of binary star systems.

Binary star systems are very important in astrophysics because calculations of their orbits allow the masses of their component stars to be directly determined, which in turn allows other stellar parameters, such as radius and density, to be indirectly estimated.



Types of binaries





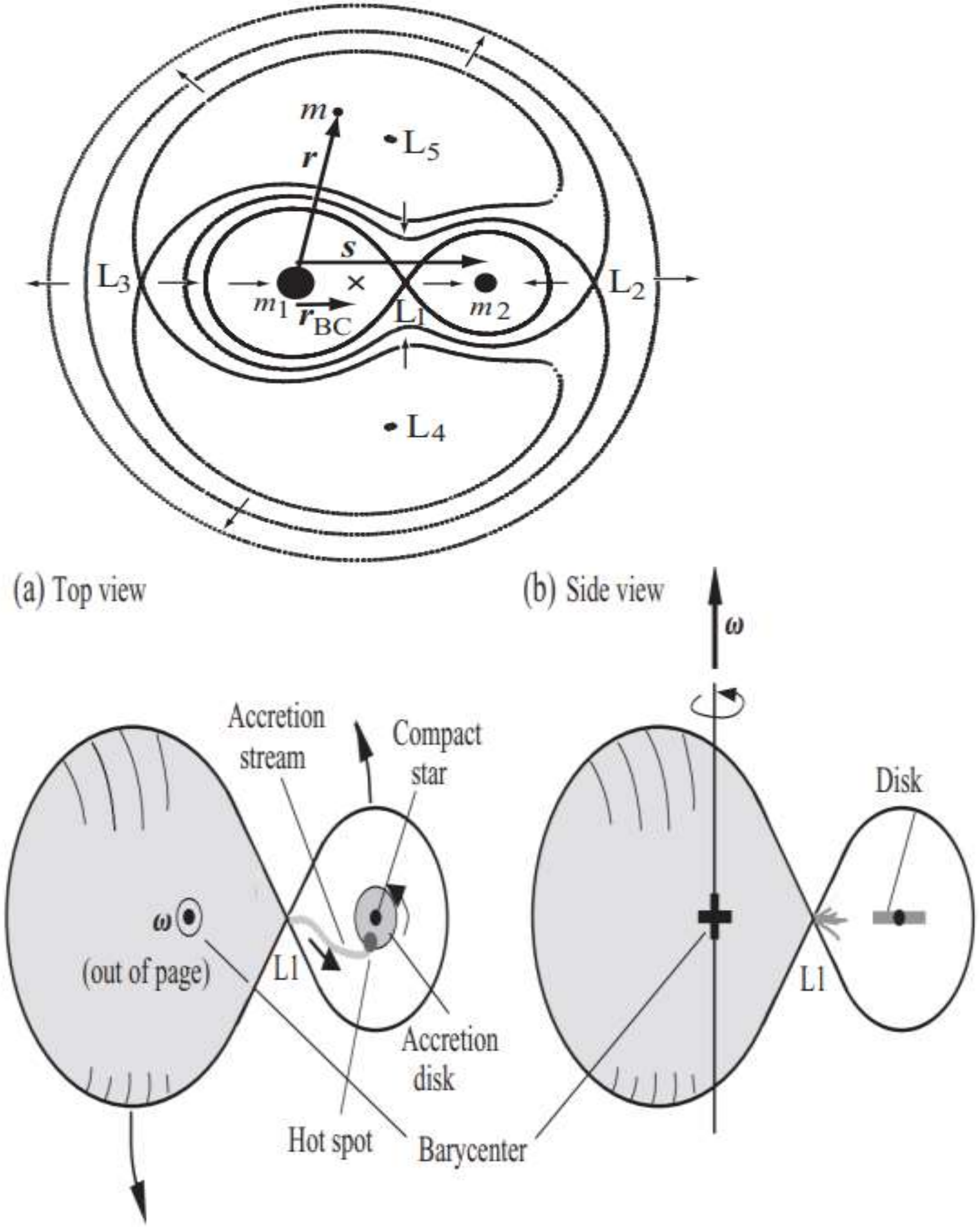
Roche lobe

The teardrop-shaped regions of space bounded by this particular equipotential surface are called Roche lobes.

$$R_{\mathrm{L,m_1}} = \frac{0.49\,q^{2/3}}{0.6\,q^{2/3} + \ln(1 + q^{1/3})}s. \qquad \text{(Radius of Roche lobe of } m_1\text{)}$$

$$q \equiv m_1/m_2,$$

Accretion



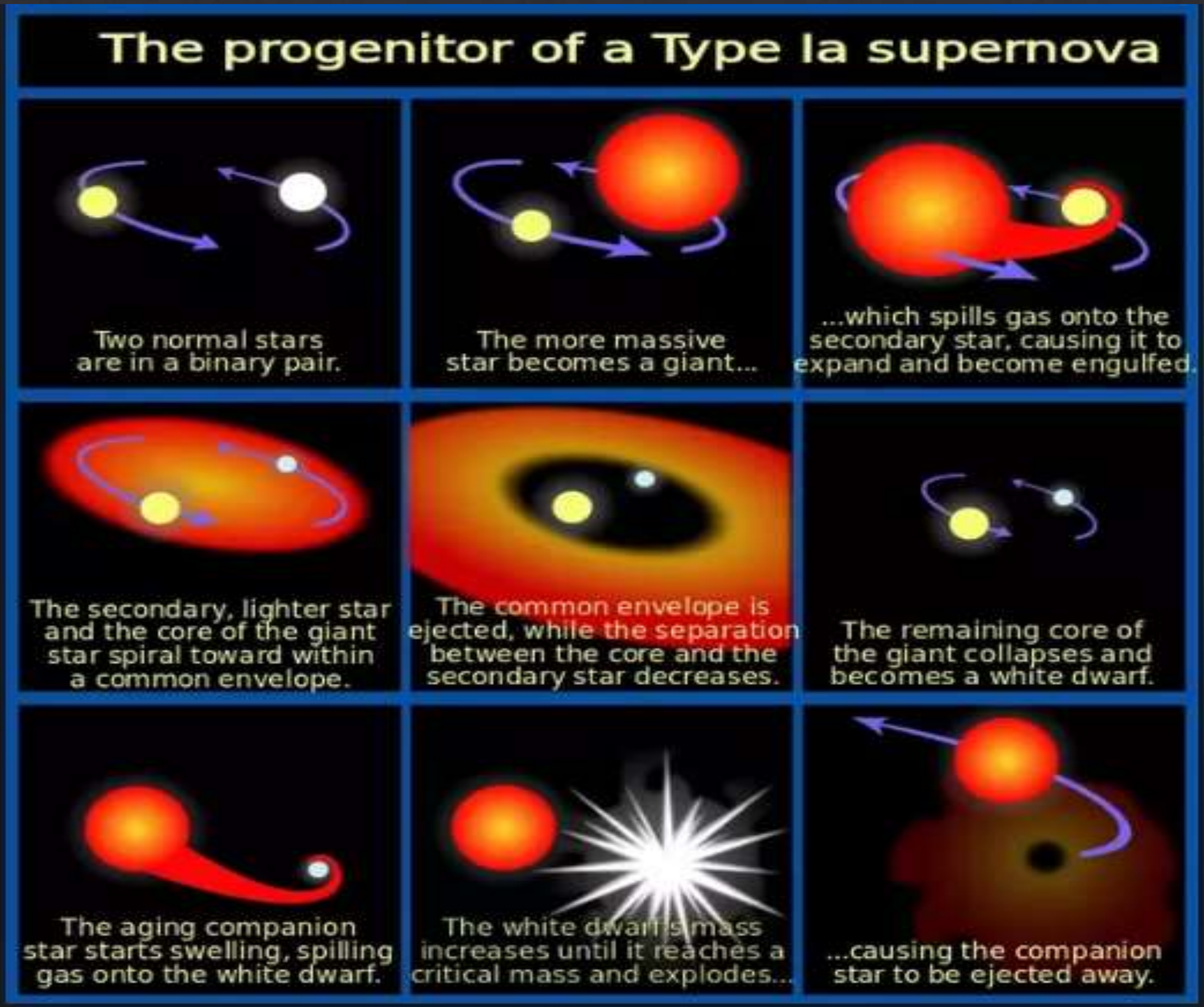
Overflowing Roche lobe. The matter from the filled lobe of the more massive star (left) exits through the L1 point and forms a counterclockwise rotating accretion disk surrounding the compact object

The two stars in a binary system can directly affect each other's evolution through accretion of matter from one to the other, tidal forces, and gravitational radiation, among other factors. The star separation and period of the orbit will be modified by the transfer of mass.

$$J = \left(\frac{G}{M_T}\right)^{1/2} m_1 m_2 s^{1/2},$$ **angular momentum**

$$M_T = m_1 + m_2, \quad s = r_2 - r_1.$$

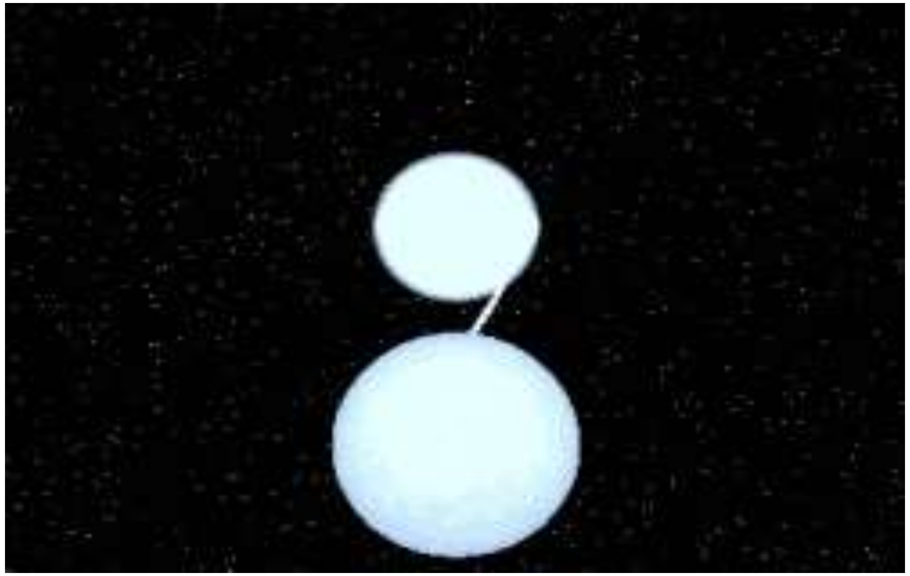
Evolution of Binary system

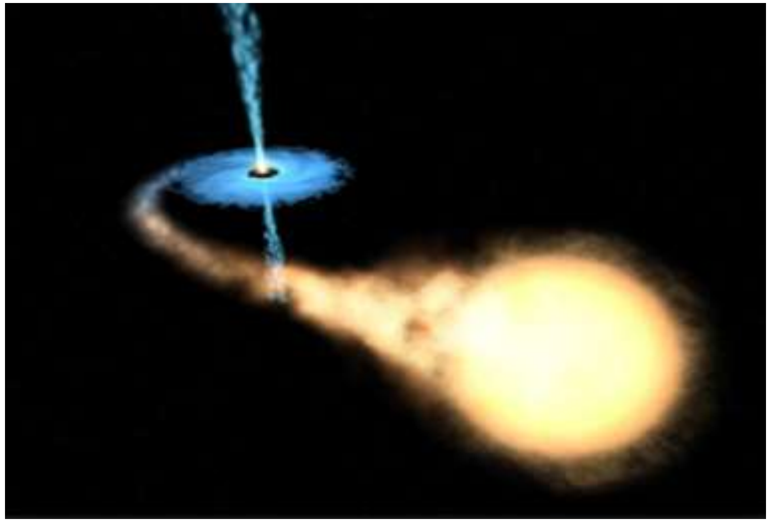


The two stars in a binary system can directly affect each other's evolution through accretion of matter from one to the other, tidal forces, and gravitational radiation, among other factors. The evolution of a particular system may thus be somewhat uncertain.

Stars in a binary system will interact tidally. Within a few stellar radii, the forces can be very strong. Such interactions result in a loss of energy that leads to *circularization* of an elliptical orbit and also to *synchronization* of the spin periods of the stars and the orbital period.

The circularization and synchronization are due to the gravitational effect of tidal bulges raised on one or both stars. When the orbit is circular and both stars are synchronized to the orbit, there is no more energy loss due to tidal dissipation. The system can then rotate with stable period and separation until some other effect perturbs it – perhaps the evolution of one of the stars





White dwarf binarys

$$\frac{R}{R_\odot} \approx 0.01 \left(\frac{M}{M_\odot}\right)^{-1/3} \qquad \text{(Radius of white dwarf star)}$$

$$\frac{\rho G M}{R^2} \sim \frac{M^2}{R^5},$$

If the white dwarf accretes matter from a binary companion, its mass will gradually increase. This will cause it to decrease slowly in size

$$M_{\text{Ch}} = 1.46 \left(\frac{2}{\mu_e}\right)^2 M_\odot \underset{\mu_e=2}{\rightarrow} 1.46\, M_\odot, \qquad \text{(Chandrasekhar mass limit)}$$

Main sequence star and black hole

the black hole is evident to us because of the substantial gaseous matter being accreted into it. A possible evolutionary path of two normal stars to a high-mass x-ray binary

Gravitational radiation

As a first step toward the understanding of GWs, we wish to study the expansion of the Einstein equations around the flat-space metric. Therefore we write $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ $|h_{\mu\nu}| \ll 1$

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}(\partial_\rho\partial_\nu h_{\mu\sigma} + \partial_\sigma\partial_\mu h_{\nu\rho} - \partial_\sigma\partial_\nu h_{\mu\rho} - \partial_\rho\partial_\mu h_{\nu\sigma})$$

The Riemann Curvature Tensor becomes

$$R_{\mu\nu} = \frac{1}{2}\left(\partial_\sigma\partial_\nu h^\sigma{}_\mu + \partial_\sigma\partial_\mu h^\sigma{}_\nu - \partial_\mu\partial_\nu h - \Box h_{\mu\nu}\right)$$

The Ricci Tensor

$h = \eta^{\mu\nu}h_{\mu\nu}$ is the trace of the perturbation

$\Box = -\partial_t^2 + \nabla^2$ d'Alambertian operator.

$$R = \partial_\mu\partial_\nu h^{\mu\nu} - \Box h$$

$$G_{\mu\nu} = \frac{1}{2}\left(\partial_\sigma\partial_\nu h^\sigma{}_\mu + \partial_\sigma\partial_\mu h^\sigma{}_\nu - \partial_\mu\partial_\nu h - \Box h_{\mu\nu} - \eta_{\mu\nu}\partial_\rho\partial_\lambda h^{\rho\lambda} + \eta_{\mu\nu}\Box h\right)$$

The Einstein Tenzor

$$\bar{h}^{\mu\nu} \equiv h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}h \qquad G = c = 1$$

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$\partial_\nu \bar{h}^{\mu\nu} = 0$$

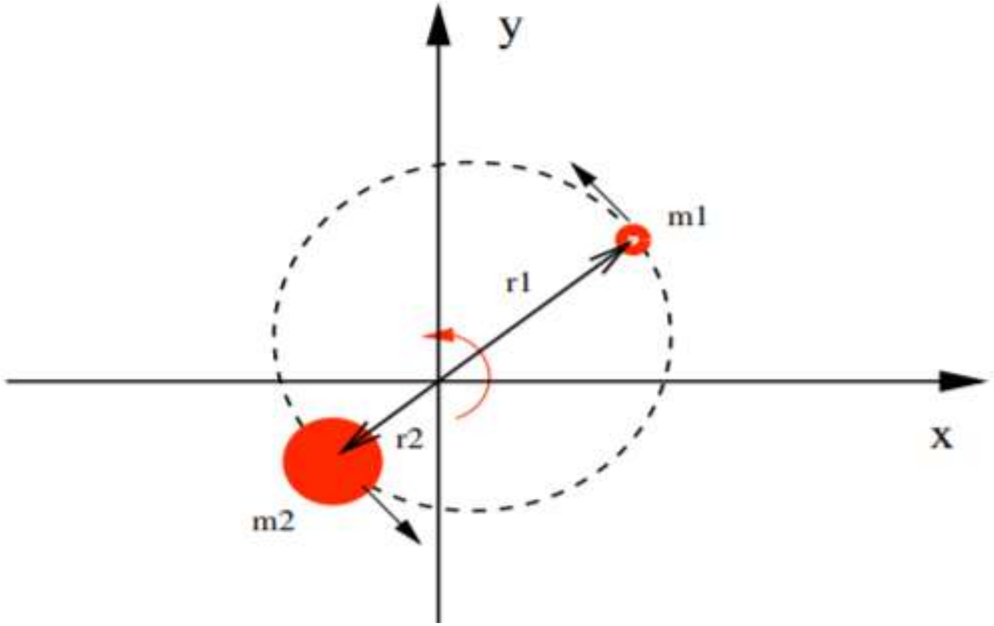
$$\Box \bar{h}^{\mu\nu} = -16\pi\, T^{\mu\nu}$$

This is the wave equation for the perturbation, and by solving it for a specific gauge and different values of $T^{\mu\nu}$ we will derive interesting results concerning gravitational waves in Nearly Lorentz coordinate systems.

Our first interesting result on gravitational waves is obtained by solving (4.17) in vacuum, i.e., $T^{\mu\nu} = 0$. The wave equation becomes homogeneous

$$\left(-\partial_t^2 + \nabla^2\right) \bar{h}^{\mu\nu} = 0 \tag{4.18}$$

$$\bar{h}_{\mu\nu}(\omega, r) = \frac{A_{\mu\nu}(\omega)}{r}\, e^{i\frac{\omega}{c}r}.$$



$$M \equiv m_1 + m_2,$$ M the total mass

$$\mu \equiv \frac{m_1 m_2}{M}.$$ μ the reduced mass

$$G\frac{m_1 m_2}{l_0^2} = m_1\, \omega_K^2 \frac{m_2 l_0}{M}, \qquad G\frac{m_1 m_2}{l_0^2} = m_2\, \omega_K^2 \frac{m_1 l_0}{M},$$

The orbital frequency can be found from Kepler's law

$$\omega_K = \sqrt{\frac{GM}{l_0^3}}$$

$$x_1 = \tfrac{m_2}{M} l_0 \cos\omega_K t \qquad x_2 = -\frac{m_1}{M} l_0 \cos\omega_K t$$

$$y_1 = \tfrac{m_2}{M} l_0 \sin\omega_K t \qquad y_2 = -\frac{m_1}{M} l_0 \sin\omega_K t.$$

The corresponding energy density is:

$$T^{00} = c^2 \sum_{n=1}^{2} m_n\ \delta(x - x_n)\ \delta(y - y_n)\ \delta(z)\,,$$

the solution of the wave equation inside the source gives the wave amplitude Aμν (ω) as an integral of the stress-energy tensor of the source over the source volume

$$A_{ij}(t) = \begin{pmatrix} \cos\ 2\omega_K t & \sin\ 2\omega_K t & 0 \\ \sin\ 2\omega_K t & -\cos\ 2\omega_K t & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{\mu\nu}(\omega) = \frac{4G}{c^4} \int_V T_{\mu\nu}(\omega, x^i)\ d^3x.$$

$$\bar{h}_{\mu\nu}(\omega, r) = \frac{A_{\mu\nu}(\omega)}{r}\ e^{i\frac{\omega}{c}r}.$$

$$\bar{h}_{\mu\nu}(\omega, r) = \frac{4G}{c^4} \cdot \frac{e^{i\ \omega\frac{r}{c}}}{r} \int_V T_{\mu\nu}(\omega, x^i)\ d^3x,$$

we shall use the conservation law that Tμν satisfies

$$\frac{\partial T^{\mu\nu}}{\partial x^\nu} = 0, \qquad \rightarrow \qquad \frac{1}{c}\frac{\partial T^{\mu 0}}{\partial t} = -\frac{\partial T^{\mu k}}{\partial x^k}, \qquad \mu = 0..3, \quad k = 1..3.$$

$$q^{kn}(t) = \frac{1}{c^2}\int_V T^{00}(t, x^i)\, x^k\, x^n d^3x, \qquad k, n = 1, 3,$$

quadrupole moment tensor of the system

$$q_{xx} = \frac{\mu}{2}\, l_0^2 \,\cos 2\omega_K t + cost$$
$$q_{yy} = -\frac{\mu}{2}\, l_0^2 \,\cos 2\omega_K t + cost1$$
$$q_{xy} = \frac{\mu}{2}\, l_0^2 \,\sin 2\omega_K t,$$

the non vanishing components of the quadrupole moment

from $$\bar{h}_{\mu\nu}(t, r) = \frac{4G}{c^4}\frac{1}{r}\int_V T_{\mu\nu}(t - \frac{r}{c}, x^i)\, d^3x.$$ **we finally find**

$$\bar{h}^{\mu 0} = 0, \qquad \mu = 0..3$$

$$\bar{h}^{ik}(t, r) = \frac{2G}{c^4 r}\cdot\left[\frac{d^2}{dt^2}\, q^{ik}(t - \frac{r}{c})\right]$$

This is the gravitational wave emitted by a gravitating system evolving in time. It can be composed of masses or of any form of energy, because mass and energy are both sources of the gravitational field.

Gravitational radiation has a quadrupolar nature. A system of accelerated charged particles has a time-varying dipole moment

$\frac{G}{c^4} \sim 8 \cdot 10^{-50}\ s/g\ cm$ **GW are extremely weak!**

Gravitational radiation has a quadrupolar nature. A system of accelerated charged particles has a time-varying dipole moment

$$\vec{d}_{EM} = \sum_i q_i \vec{r}_i$$

and it will emit dipole radiation, the flux of which depends on the second time derivative of d_{EM} For an isolated system of masses we can define a gravitational dipole moment

$$\vec{d}_G = \sum_i m_i \vec{r}_i,$$

which satisfies the conservation law of the total momentum of an isolated system

$$\frac{d}{dt}\vec{d}_G = \vec{0}.$$

For this reason, gravitational waves do not have a dipole contribution. It should be stressed that for a spherical or axisymmetric, stationary distribution of matter (or energy) the quadrupole moment is a constant, even if the body is rotating. Thus, a spherical or axisymmetric star does not emit gravitational waves; similarly a star which collapses in a perfectly spherically symmetric way has a vanishing q_{ik} and does not emit gravitational waves. To produce these waves we need a certain degree of asymmetry, as it occurs for instance in the non-radial pulsations of stars, in a non spherical gravitational collapse, in the coalescence of massive bodies etc.

the orbital period T of a binary system decreases in time due to gravitational wave emission. $\dot{T} \equiv dT/dt$
The orbital evolution driven by gravitational wave emission quietly proceeds bringing the stars closer. As their distance decreases, the process becomes faster and the two stars spiral toward their common center of mass until they coalesce.

$$L_{GW} \equiv \frac{dE_{GW}}{dt} = \frac{32}{5}\frac{G^4}{c^5}\frac{\mu^2 M^3}{l_0^5}.$$

the gravitational luminosity
Power radiated in gravitational waves by a binary system in circular orbit:

$$\frac{dP}{dt} = \frac{3}{2}\frac{P}{E_{orb}} L_{GW}.$$

how the orbital period changes due to the emission of gravitational waves

the orbit of the real system has a quite strong eccentricity ! 0.617. By doing the calculation using the equations of motion appropriate for an eccentric orbit we would find

$$\frac{dP}{dt} = -2.4 \cdot 10^{-12}.$$

PSR 1913+16 has now been monitored for decades and the rate of variation of the period, measured with very high accuracy, is

$$\frac{dP}{dt} = -(2.4184 \ \pm 0.0009) \cdot 10^{-12}.$$

Thus, the prediction of General Relativity are confirmed by observations. This result provided the first indirect evidence of the existence of gravitational waves and for this discovery Hulse and Taylor have been awarded of the Nobel prize in 1993. For the recently discovered double pulsar PSR J0737-3039

$$P = 8640\ s, \qquad E_{orb} \sim -2.55 \cdot 10^{48}\ erg, \qquad L_{GW} \sim 2.24 \cdot 10^{32}\ erg/s$$

$$\boxed{\frac{\dot{P}_{corrected}}{\dot{P}_{GR}} = 1.0013(21)}$$

Knowing the energy lost by the system, we can also evaluate how the orbital separation l0 changes in time

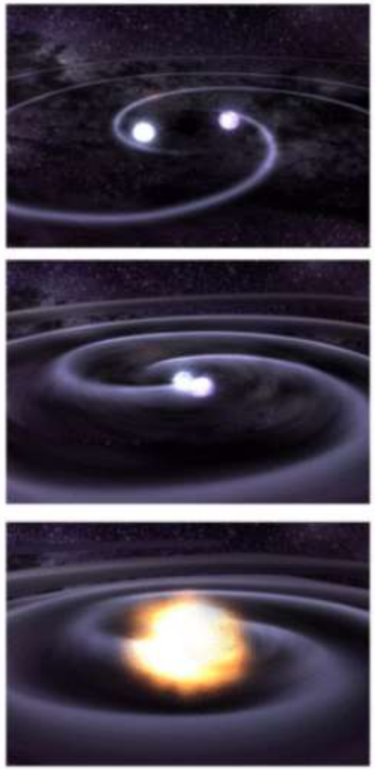
$$l_0(t) = l_0^{in}\left[1 - \frac{t}{t_{coal}}\right]^{1/4}.$$

$$l_0(t = 0) = l_0^{in} \qquad t_{coal} = \frac{5}{256}\frac{c^5}{G^3}\frac{(l_0^{in})^4}{\mu M^2},$$

which shows that the orbital separation decreases in time.

From this equation we see that when t = tcoal the orbital separation becomes zero, and this is possible because we have assumed that the bodies composing the binary system are pointlike. Of course, stars and black holes have finite sizes, therefore they start merging and coalesce before t = tcoal is reached. In addition, when the two stars are close enough both the slow motion approximation and the weak field assumption on which the quadrupole formalism relies fails to hold and strong field effects have to be considered; however, the value of tcoal gives an indication of the time the system needs to merge starting from a given initial distance l in 0 .

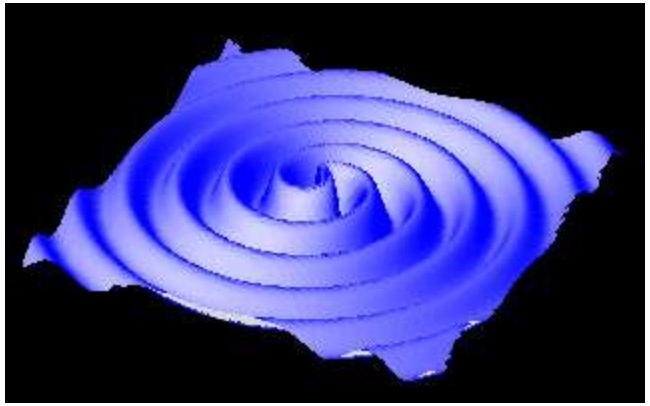
So, two masses orbiting one another will radiate gravitational waves according to GR because the mass distribution has a time varying quadrupole moment. The energy loss due to this has been dramatically demonstrated through the observation of the orbital decay of a two neutron-star binary system .



$$\sim m\frac{da}{dt}.$$ amplitude

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{64}{5}\frac{G^3}{c^5}\frac{(m_1 m_2)(m_1+m_2)}{r^3},$$

where r is the separation between the bodies, t time, G the gravitational constant, c the speed of light, and m_1 and m_2 the masses of the bodies



$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{32}{5}\frac{G^4}{c^5}\frac{M_T^3\mu^2}{s^5}$$ (Gravitational energy loss;

$$\underset{m_1=m_2\equiv m}{\rightarrow} -\frac{64}{5}\frac{G^4}{c^5}\left(\frac{m}{s}\right)^5,$$ circular orbits; W)

$\mu = m_1 m_2/M_T$ is the reduced mass $M_T = m_1 + m_2$ the total mass. **S the star separation**

Note that large masses and small separations greatly increase the rate of energy loss.

References:

1. H. Bradt, “Astrophysics Processes”, (2008)
2. Michele Maggiore, Gravitational Waves (2008)
3. Valeria Ferrari, “Sapienza”, Universit`a di Roma THE QUADRUPOLE FORMALISM APPLIED TO BINARY SYSTEMS (2010)
4. R. W. HILDITCH An Introduction to Close Binary Stars (2001)
5. Stuart L. Shapiro, Saul A. Teukolsky : “Black Holes, White Dwarfs and Neutron Stars: The Physics of Compact Objects” (1983)
6. А.М.Фридманб Л.В.Бисикалою Природа аккреционных дисков тесных двойных звездЖнеустойчивасть сверхотражение и развитая турбулентность(2008)
7. A.C. Phillips “The physics of stars”.

Thank you for your attention