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Compact Objects

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- White dwarfs
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- Black Holes

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What Are Compact Objects

Compact objects - white dwarfs, neutron stars, and black holes - are "born" when normal stars "die," that is, when most of their nuclear fuel has been consumed. **They are the end products of stellar evolution.**

Compact objects differ from normal stars in two fundamental ways:

i. Since they do not burn nuclear fuel, they cannot support themselves against gravitational collapse by generating thermal pressure.
   - White dwarfs - pressure of degenerate electrons.
   - Neutron stars – (largely) pressure of degenerate neutrons.
   - Black holes - completely collapsed stars (They could not find any means to hold back the inward pull of gravity and therefore collapsed to singularities).

ii. Relative to normal stars of comparable mass, compact objects have much smaller radii and hence, much stronger surface gravitational fields.

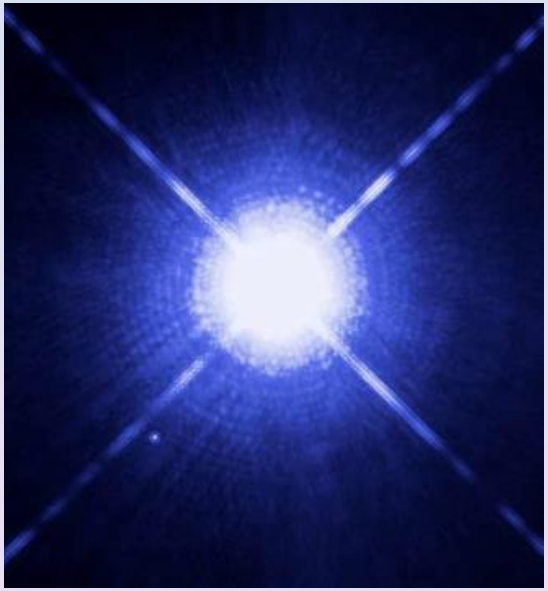
Their analysis requires a deep physical understanding. All four fundamental interactions play a role in compact objects (Even for white dwarfs, where Newtonian gravitation is adequate to describe their equilibrium structure, general relativity turns out to be important for a proper understanding of their stability).

The Formation of Compact Objects

The primary factor determining whether a star ends up as a white dwarf, neutron star, or black hole is thought to be the star's mass.

❖ White dwarfs are believed to originate from light stars with masses $M \leq 4\, M_{\circledast}$.

$$R_w \sim 0.008 - 0.02\, R_{\odot}$$
$$n \sim 10^{25} - 10^{29}\ \text{cm}^3$$
$$B \sim 10^5 - 10^9\ \text{G}$$
$$T_s < 150\,000\ K$$



Sirius-B (Hubble Space Telescope , 1862):

M - 0.97 (1.05)$M_{\circledast}$;   R - 0.0084 $R_{\circledast}$; ρ - 2.5 $\cdot 10^9\ kg/m^3$

❖ Neutron stars and black holes are believed to originate from more massive stars. However, the dividing line between those stars that form neutron stars and those that form black holes is very uncertain because the final stages of evolution of massive stars are poorly understood.

The Formation of Compact Objects

Total gravitational collapse leading to a black hole can occur routes other than by the direct collapse of an evolved, massive star.

For example since there is a definite maximum mass above which a white dwarf or a neutron star can no longer support itself against collapse, the accretion of gas by either these objects can lead to black hole formation.

Two additional black hole formation processes have been proposed by theoreticians :

i. The collapse of a hypothetical "supermassive star," which leads to the formation of a **"supermassive black hole."** Supermassive stars are known to be unstable when they reach a certain critical density, depending on mass.

ii. The formation of **primordial black holes** in the early Universe, due to perturbations in the homogeneous background density field.

Hydrostatic equilibrium

To be stable a star must be in mechanical equilibrium. (**A star is stable when its internal gas pressure matches its gravitational pressure**).

$$P_g = \frac{F_g}{4\pi r^2} = -\frac{2}{3}\pi G \rho^2 R^2$$

P_g - the gravitational pressure , F_g - the gravitational force.

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} \rightarrow \rho^2 = \frac{9}{16}\frac{M^2}{\pi^2 R^6}$$

$$\Rightarrow P_g = -\frac{3}{8\pi} G \frac{M^2}{R^4}.$$

Normal star follows the ideal gas law: $P_i \approx \mathrm{n}kT \approx \frac{MkT}{\frac{4}{3}\pi R^3 m_H}.$

For hydrostatic equilibrium to hold: $P_i = -P_g.$

$$M = \frac{2RkT}{Gm_H}.$$

M(R) relation is a consequence of hydrostatic equilibrium.

Hydrostatic equilibrium

If nuclear reactions were suddenly turned off, the star would begin to collapse and generate its energy by gravitational contraction. After some time we would expect the star to become sufficiently dense that gas degeneracy would begin to play a role.

- **At what radius that might happen?**

The degeneracy condition for electrons: $n \gg 5 \times 10^{15} T^{3/2} \rightarrow n > n_c = 5 \times 10^{16} T^{3/2}$

- the number density, n_c - critical density (above which a gas is considered fully degenerate).

For $T = 10^7$ K, $n_c \approx 10^{27}$ cm^{-3}. But

$$n = \frac{M}{\frac{4}{3}\pi m_H R^3}$$

so that for $M = 1\,M_\odot$ and $m_H \approx 10^{-24}$ g

$$R < R_c \approx 10^9 \text{ cm} = 10^4 \text{ km}.$$

New hydrostatic equilibrium based on the **degeneracy pressure.**

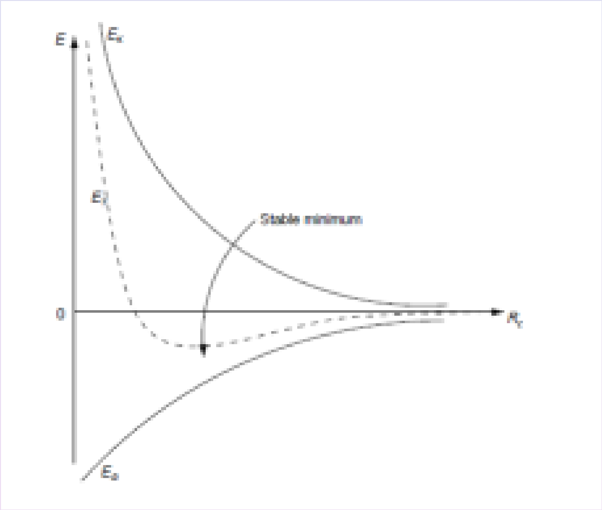
Stability of compact stars

The total energy of a star is given by the energy conservation law:

$$E_T = E_G + E_k = -\frac{GM^2}{R_c} + UV.$$

For non-relativistic gas:

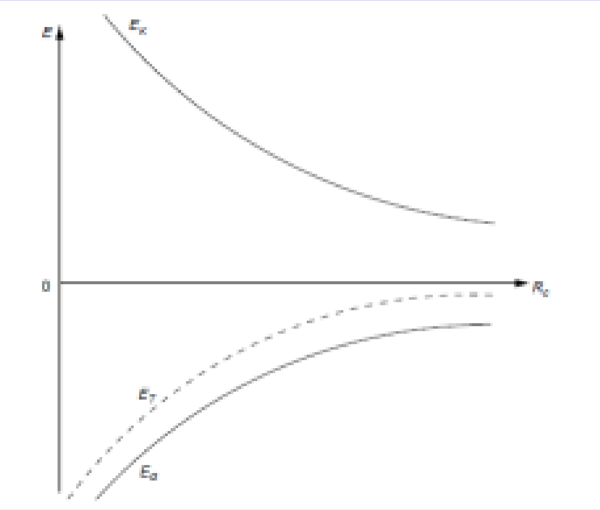
$$E_T = -\frac{E_{G0}}{R_c} + \frac{E_{k0}}{R_c^2}$$



- Energy curve showing a stable point. When the total energy curve has a local minimum with respect to R the star has a stable point where it can exist in hydrostatic equilibrium.

In relativistic case:

$$E_T = -\frac{E_{G0}}{R_c} + \frac{E_{k0}}{R_c}$$



- Energy curve with no stable points. When the energy curve has no local minimum the star cannot attain hydrostatic equilibrium and it will collapse.

White dwarfs

White dwarf equation of state:

$$P = \begin{cases} (1/8)(3c^3h^3/\pi)^{1/3}n^{4/3} & \text{(relativistic)} \\ (1/(10m))(3h^3/8\pi)^{2/3}n^{5/3} & \text{(non-relativistic).} \end{cases}$$

Mass–radius relation for white dwarfs:

$$M = \left[\frac{8\pi}{30Gm_e}\right]^3 \left[\frac{3h^3}{8\pi}\right]^2 \left[\frac{3}{4\pi m_H}\right]^5 R^{-3} \approx 10^{60} R^{-3} \qquad \text{(non-relativistic).}$$

$$M \approx \left(\frac{2\pi ch}{3G}\right)^{3/2} \left(\frac{3}{4\pi m_H}\right)^2 \approx 10^{34} \qquad \text{(relativistic).}$$

Radius of white dwarf star:

$$\frac{R}{R_\odot} \approx 0.01 \left(\frac{M}{M_\odot}\right)^{-1/3}.$$

A white dwarf of one solar mass is thus expected to have a radius about 1% that of the sun in accord with observations.

Note that the stellar **radius decreases with increasing mass.** In contrast, on the main sequence, massive stars are larger than less massive stars.

White dwarfs

The EOS of nonrelativistic degenerate gases leads to two interesting phenomena pertaining to WD's.

i. The first is that **degeneracy pressure of electrons can support a star against collapse** even if its nuclear fuel has been exhausted.

ii. The second is that, if a white dwarf is more **massive than $\sim 1.4\ M_{\circledast}$**, the electrons become relativistic and the **EOS becomes softer.** In this situation, degeneracy pressure is no longer able to support the star.

If the white dwarf accretes matter from a binary companion, its mass will gradually increase. This will cause it to decrease slowly in size. At this point, the EOS will approach the limiting EOS for totally relativistic electrons.

$$\rho^{\frac{5}{3}} \rightarrow \rho^{\frac{4}{3}} \Rightarrow$$ ***The stable equilibrium has been lost.***

White dwarfs

Chandrasekhar mass limit:

The mass at which the collapse would occur, namely ~1.4 $M_{\circledast}$, is known as the Chandrasekhar mass limit:

$$M_{\mathrm{Ch}} = 1.46 \left(\frac{2}{\mu_e} \right)^2 M_\odot \underset{\mu_e = 2}{\longrightarrow} 1.46\, M_\odot,$$

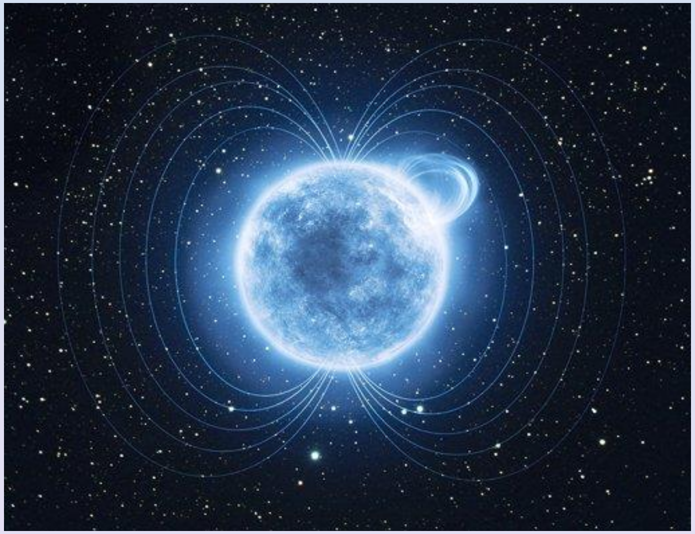
The result is named after the young Indian physicist who first calculated it in 1932; he was awarded the Nobel prize for this work in 1983.

Relativistic stars are unstable because the pressure does not increase fast enough with density to achieve hydrostatic equilibrium. Physically, on the microscopic scale, the relativistic electrons combine with the protons to form neutrons thereby reducing the available electrons for pressure support. The neutron production leads to the instability of relativistic stars.

A collapsing white dwarf must stabilize itself as a neutron star if it is to avoid gravitational collapse into a black hole.

Neutron stars

Neutron stars have very ***high density and small radius***, and are much more gravitationally bound than ordinary stars.



M < 3.6$M_{\circledast}$ R ~ 20 km B ~10^8-10^{17} G

$\mathrm{T}_s \sim 6 \times 10^5\ K$ $\mathrm{T}_n \sim 10^{11} - 10^{12}\ K$

ρ_s ~10^9 kg/m^3 $\rho_n \sim 10^{17}$ kg/m^3

The ***surface gravity*** is given:

$$g_N = \frac{GM}{R^2} \approx 10^{14}\ \mathrm{cm\ s^{-2}}.$$

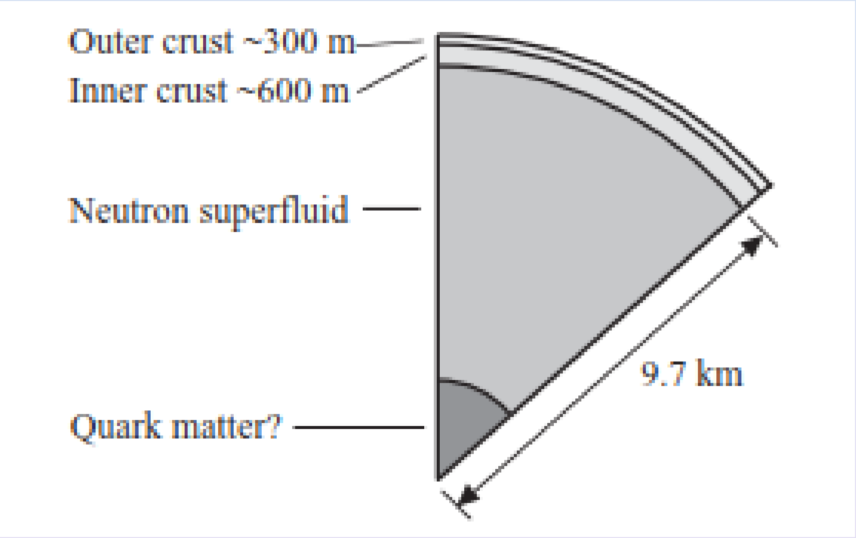
Compare that to the Earth where g ≈ 10^3cm s^{-3}.

They result from the supernova explosion of a massive star (which before collapse had a total mass of between 10 $M_{\circledast}$ and 29 $M_{\circledast}$), combined with gravitational collapse, that compresses the core past WD star density to that of atomic nuclei.

Pulsars (neutron stars) are rotating compact remnants (T = 10^{-3} -1 s) of the supernova explosion with extremely powerful magnetic.

Neutron stars

The several levels in the star are



a) A gaseous atmosphere less than 0.1-m thick (not shown);
b) A thin outer, electrically conductive crust containing nuclei in a rigid lattice together with relativistic degenerate electrons about 300 m thick;
c) An inner crust about 600-m thick consisting of a lattice of neutron-rich nuclei, a relativistic electron gas, and (probably superfluid) neutrons;
d) A region of neutron liquid (a superfluid); which may also contain some electrons and superfluid protons;
e) A core of uncertain nature – possibly some exotic material such as quark matter.

Neutron stars

Radius of a neutron star:

$$\rho_n = \frac{m_n}{\frac{4}{3}\pi r_n^3} = 1.2 \times 10^{17}\,\mathrm{kg/m^3}, \qquad \text{(Density of a neutron)}$$

(where density ρ_n of a single neutron of radius r_n and mass m_n),where we use m_n = 1.7 × 10^{27}kg and r_n= 1.5 × 10^{15}m.

$$M_{ns} = \frac{4}{3}\pi R_{ns}^3 \rho_{ns}$$

If M_{ns} = 1.4 M$M_{\circledast}$, the approximate radius of a neutron star as follows:

$$R_{ns} \approx \left(\frac{3\,M_{ns}}{4\,\pi\,\rho_{ns}}\right)^{1/3} = 9.3 \times 10^3\,\mathrm{m} \approx 10\,\mathrm{km}.$$

This 10-km radius is the nominal size often attributed to a neutron star. As with normal stars, the size is expected to be a function of the mass of the neutron star. Sizes of ~10 km have also been measured through the luminosity-temperature dependence of x-ray bursts, which are characterized by rapid nuclear burning of accreted material on the surface of a neutron star.

Neutron stars

If the neutron star has a sufficiently strong magnetic field ,the gas will be guided by the magnetic field onto localized regions of the star, which become x-ray hot and perhaps the dominant *source of x rays*. These hot spots come into and out of view as the neutron star rotates, thus giving rise to pulses of x rays.

Maximum mass:

At some sufficiently large mass, the nuclear and degeneracy pressures could no longer withstand the inward pull of gravity, the ensuing collapse would nevitably carry the entire star to within the Schwarzschild radius to become a black hole. There is no known force that could withstand gravitational forces of this magnitude.

Consider a hypothetical neutron star of uniform density:

$$R \approx \left(\frac{M}{4\rho}\right)^{1/3}$$

As mass is added to a neutron star, the R_s increases linearly with M and eventually becomes greater than the neutron star radius. At this point, the neutron star will become a black hole.

From the concept of the Schwarzschild radius $R_s = 2GM/c^2$

$$\frac{2GM}{c^2} > \left(\frac{M}{4\rho}\right)^{1/3}. \qquad \text{(Condition for neutron-star collapse)}$$

Mass limits in the range: $1.46M_{\circledast}$<M < $3.6M_{\circledast}$

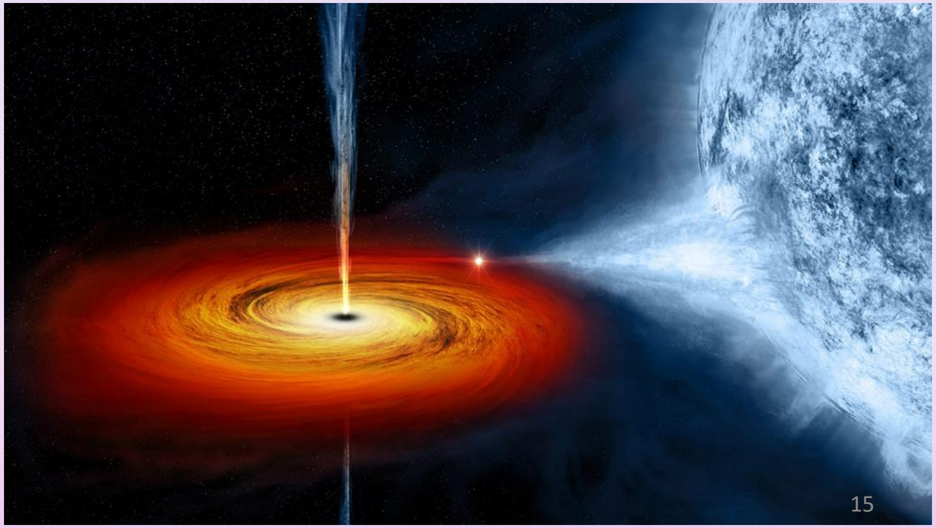
Black Holes

We have seen that both white dwarfs and neutron stars have a maximum possible mass.

- What happens to a neutron star that accretes matter and exceeds the mass limit?
- What is the fate of the collapsing core of a massive star, if the core mass is too large to form a neutron star?

According to general relativity, is that nothing can halt the collapse. As the collapse proceeds, the gravitational field near the object becomes stronger and stronger. Eventually, nothing can escape from the object to the outside world, not even light.

A black hole has been born.



Black Holes

A black hole is defined simply as a region of spacetime that cannot communicate with the external universe. The boundary of this region is called the surface of the black hole, or the ***event horizon.***

- ❑ Problem: If we extrapolate Einstein's equations inside a black hole, then a singularity develops.

As long as the singularity is hidden inside the event horizon, it cannot influence the outside world. So we can continue to use general relativity to describe the observable universe.

Black holes can be formed from stars with varying mass distributions, shapes (multipole moments), magnetic field distributions, angular momentum distributions, and so on. Remarkably, the most general stationary black hole solution is known analytically. It depends on only three parameters:

- The mass M
- Angular momentum J
- Charge Q

Black Holes

Schwanschild Black Holes :

- Vacuum solution of Einstein's field equations, it is valid only in the empty space outside the object.

$$R_{\mu\nu} = 0$$

The Schwarzschild solution:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2\, dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2\, d\Omega^2$$

Schwarzschild solution is called a black hole, since nothing can escape from the horizon.

$$R_S = \frac{2GM}{c^2}. \qquad \text{(Schwarzschild radius or event horizon for black hole)}$$

A mass of ~6 $M_{\odot}$ of radius less than 18 km would be a black hole.
For earth R_S*=9mm.*

❖ ***A black hole is a region of an asymptotically flat spacetime from which nothing can escape, not even light.***

Black Holes

- What happens to an object that falls into a nonrotating black hole?

The theory indicates it will continue to fall inward, reaching a central point ***the Schwarzschild singularity.***
Lets now estimate the size scale at which quantum effects become important. This occurs when the radius of the event horizon, ~GM/c^2, for a given mass is so small that it matches the Compton wavelength of that mass;

$$\frac{Gm}{C^2} = \frac{\hbar}{mc},$$

$$m_P = \left(\frac{\hbar c}{G}\right)^{1/2} = 2.2 \times 10^{-8}\,\text{kg.} \qquad \text{(Planck mass)}$$

$$\ell_P = \frac{\hbar}{m_P c} = \left(\frac{\hbar G}{c^3}\right)^{1/2} = 1.6 \times 10^{-35}\,\text{m.} \qquad \text{(Planck length)}$$

The region where quantum effects must enter.

$$t_P = \frac{\ell_P}{c} = \left(\frac{\hbar G}{c^5}\right)^{1/2} = 5.4 \times 10^{-44}\,\text{s.} \qquad \text{(Planck time)}$$

There is no way to discuss times shorter than the Planck time or distances smaller than the Planck length without a theory of quantum gravity.

Black Holes

Kerr Black Holes:

The most general stationary black hole metric, with parameters M , J, and Q,- Kerr-Newman metric.

- Special cases are the **Kerr metric** (Q = 0), the **Reissner-Nordstrom** metric (J = 0), and the **Schwarzschild metric** (J = 0, Q = 0).

The event horizon of a Kerr black hole :

$$R_{\rm h} = \frac{GM}{c^2}\left[1 + (1 - j^2)^{1/2}\right]. \qquad \text{(Event horizon; } j \equiv J/J_{\max}\text{)}$$

This solution of Einstein's equations, discovered by Kerr in 1963:

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right)dt^2 - \frac{4aMr\sin^2\theta}{\Sigma}\,dt\,d\phi + \frac{\Sigma}{\Delta}dr^2$$

$$+\Sigma\,d\theta^2 + \left(r^2 + a^2 + \frac{2Mra^2\sin^2\theta}{\Sigma}\right)\sin^2\theta\,d\phi^2.$$

where

$$a \equiv \frac{J}{M}, \qquad \Delta \equiv r^2 - 2Mr + a^2, \qquad \Sigma \equiv r^2 + a^2\cos^2\theta.$$

Black Holes

Hawking process:

Another (theoretical) characteristic of black holes, whether or not rotating, is that they emit a thermal spectrum of particles if quantum effects are taken into account.

The energy radiated arises from "virtual" pairs of particles that are continually being created and annihilated in the fluctuations of the vacuum. The strong gravity near a black hole can separate the charges, and so some become real particles.

One is captured by the black hole, and the other escapes (tunnels) to infinity. In optically thick conditions, the emerging particle kinetic energies are converted to radiation with a blackbody spectrum.

The **temperature of the black hole** is, according to the theory,

$$T = \frac{\hbar c^3}{8\pi k\, G M} = 6.1 \times 10^{-8} \left(\frac{M_\odot}{M}\right) \mathrm{K}, \quad \text{(Temperature of black hole)}$$

Luminosity:

$$L = 4\pi R_S^2\, \sigma T^4 \propto M^2 \times M^{-4} \propto M^{-2}$$

The luminosity increases rapidly as mass decreases.

Black Holes

Hawking process:

The entire energy radiation time:

$$\tau_c = 1.4 \times 10^{10} \left(\frac{M}{5 \times 10^{11}\,\mathrm{kg}} \right)^3 \mathrm{yr.} \quad \text{(Characteristic evaporation time)}$$

The lifetime thus shortens drastically as the mass decreases. The evaporation therefore accelerates and the black hole actually evaporates away to nothing.

In its last 0.1s, or perhaps earlier when it reaches temperature $\sim 10^{12}$ K, the photon energies become sufficient to create pi mesons; the black hole explodes in a burst of particles and gamma rays. Such bursts could, in principle, be detected with radio or gamma-ray instruments of sufficient sensitivity.

This equation indicates that a black hole of mass $\sim 5 \times 10^{11}$ kg would radiate its energy away in $\sim 10^{10}$yr, the age of the universe. It is a “mini” black hole of mass only $\sim 10^{-19}\, M_{\circledast}$.

It has been suggested that many of these *“mini” black holes* could have been created in the hot dense phases of the early universe and that these would be evaporating and exploding throughout the life of the universe.

References:

1. H. Bradt, *"Astrophysics Processes"*, (2008).
2. Stuart L. Shapiro, Saul A. Teukolsky : "*Black Holes, White Dwarfs and Neutron Stars: The Physics of Compact Objects"* (1983)
3. N. Duric *"Advanced astrophysics"* (2004).
4. A.C. Phillips *"The physics of stars".*
5. M. Camenzind *"Compact Objects in Astrophysics".*

Thank You