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Fundamental Physics

The Cosmological Constant Problem

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Vacuum energy as observable effect

Vacuum energy in QFT

- Energy spectrum of simple harmonic oscillator in quantum mechanics:

$$E_n = \left(n + \frac{1}{2}\right)\omega \, . \tag{1}$$

  Free quantum field theory is formulated as an infinite series of simple harmonic oscillators. For example vacuum energy density in free scalar field theory is

$$\langle \rho \rangle = \int_0^\infty \frac{d^3k}{(2\pi)^3}\frac{1}{2}\omega(\boldsymbol{k}) = \int_0^\infty \frac{d^3k}{(2\pi)^3}\frac{1}{2}\sqrt{\boldsymbol{k}^2 + m^2} \, . \tag{2}$$

- Spontaneous symmetry breaking gives a finite but still possibly large shift in the vacuum energy density. In this case

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \quad \text{where} \quad V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 + \epsilon_0 \tag{3}$$

  The potential is at it's minimum value for $|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$, leading to the shift in the energy density of the ground state:

$$\langle \rho \rangle = \epsilon_0 - \frac{\mu^4}{4\lambda}. \tag{4}$$

In quantum field theory the value of the vacuum energy density has no observational consequences.

Vacuum energy as observable effect

General relativity

In GR each form of energy contributes to the energy-momentum tensor $T_{\mu\nu}$, hence gravitates.

Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \tag{5}$$

Energy-momentum tensor of the vacuum:

$$T^{\mu\nu}_{\rm vac} = -\langle\rho\rangle\, g^{\mu\nu} \tag{6}$$

From Einstein's equations

$$T^{\mu\nu}_{\rm vac} = -\frac{\Lambda}{8\pi G} g^{\mu\nu} \tag{7}$$

We can define effective cosmological constant:

$$\Lambda_{\rm eff} = \Lambda + 8\pi G\,\langle\rho\rangle \tag{8}$$

Symmetries

A theory obeys **naturalness** only if all of its small parameters would lead to an enhancement of its exact symmetry group when replaced by zero.

Example:

The upper bound on the mass of the photon from terrestrial measurements of the magnetic field yields:

$$m_\gamma^2 \lesssim \mathcal{O}(10^{-50})\ \mathrm{GeV}^2\ . \tag{9}$$

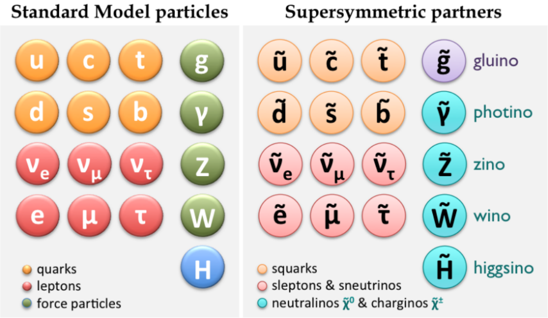
A photon with only two degrees of freedom can only get a mass if Lorentz invariance is broken.

We know that

$$\Lambda_{\mathrm{eff}} \lesssim \mathcal{O}(10^{-84})\ \mathrm{GeV}^2 \tag{10}$$

There might also be a symmetry acting to keep the effective cosmological constant so small.

Supersymmetry



Contribution to the energy of the vacuum in field theory coming from field with spin j are

$$\langle \rho \rangle = \frac{1}{2}(-1)^{2j}(2j+1)\int_0^{\lambda_{\mathrm{UV}}} \frac{d^3k}{(2\pi)^3}\sqrt{\boldsymbol{k}^2+m^2} =$$

$$= \frac{(-1)^{2j}(2j+1)}{16\pi^2}\left(\Lambda_{\mathrm{UV}}^4 + m^2\Lambda_{\mathrm{UV}}^2 - \frac{1}{4}m^4\left[\log(\frac{\Lambda_{\mathrm{UV}}^2}{m^2}) + \frac{1}{8} - \frac{1}{2}\log 2\right]\right. \qquad (11)$$

$$\left. + \mathcal{O}(\Lambda_{\mathrm{UV}}^{-1})\right)$$

Supersymmetry

If for each mass m there are an equal amount of fermionic and bosonic degrees of freedom, the net contribution to $\langle \rho \rangle$ would be zero.

$$\{Q_\alpha, \bar{Q}_\beta\} = 2\sigma^\mu_{\alpha\beta} P_\mu \tag{12}$$

For Hamiltonian we have

$$H = \frac{1}{4} \sum_\alpha \{Q_\alpha, \bar{Q}_\alpha\} = P^0 \tag{13}$$

Matrix element of P^0 can be written as

$$\langle \psi | P^0 | \psi \rangle = \tfrac{1}{4} \textstyle\sum_\alpha \langle \psi_\alpha | | \psi \alpha \rangle \quad \text{with} \quad \psi_\alpha = (Q_\alpha + \bar{Q}_\alpha) | \psi \rangle \tag{14}$$

For the vacuum $|0\rangle$ we have

$$Q_\alpha |0\rangle = 0 \quad \text{and} \quad \bar{Q}_\alpha |0\rangle = 0 \, , \tag{15}$$

thus vacuum energy is zero.

References

Nobbenhuis, S. (2006). The cosmological constant problem, an inspiration for new physics. arXiv preprint gr-qc/0609011.